Rheology of Dense Suspensions

From micro to MACRO

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IUSTI, Aix Marseille University

60th Anniversary Groupe Français de Rhéologie
October 2024

Lecture day GFR 2024 at IUSTI Laboratory

Tuesday 22th

11:00

Jean Comtet (CNRS, ESPCI Paris)

Microscopic characterization and formulation of solid interfaces

We are now here!

2:00

Francisco Melo da Rocha (IUSTI, Marseille)

Macroscopic rheological behavior of dense granular suspensions from microscopic considerations

4:00

François Peters (InPhyNi, Nice)

Numerical simulations of dense granular suspensions

Outline

Initial** (but not mandatory) **Plan:

1. *General Introduction*

2. *Granular Suspensions: The Newtonian behaviour*

   i) *Suspension as an effective fluid — Suspension Viscosity*

   ii) *Characterising the Viscosity: Volume— vs Pressure—imposed Rheology*

   iii) *Link to microstructure?*

3. *Non—Newtonian Behaviours: e.g. Shear—thickening in non-Brownian suspensions*

   i) *Experimental observations*

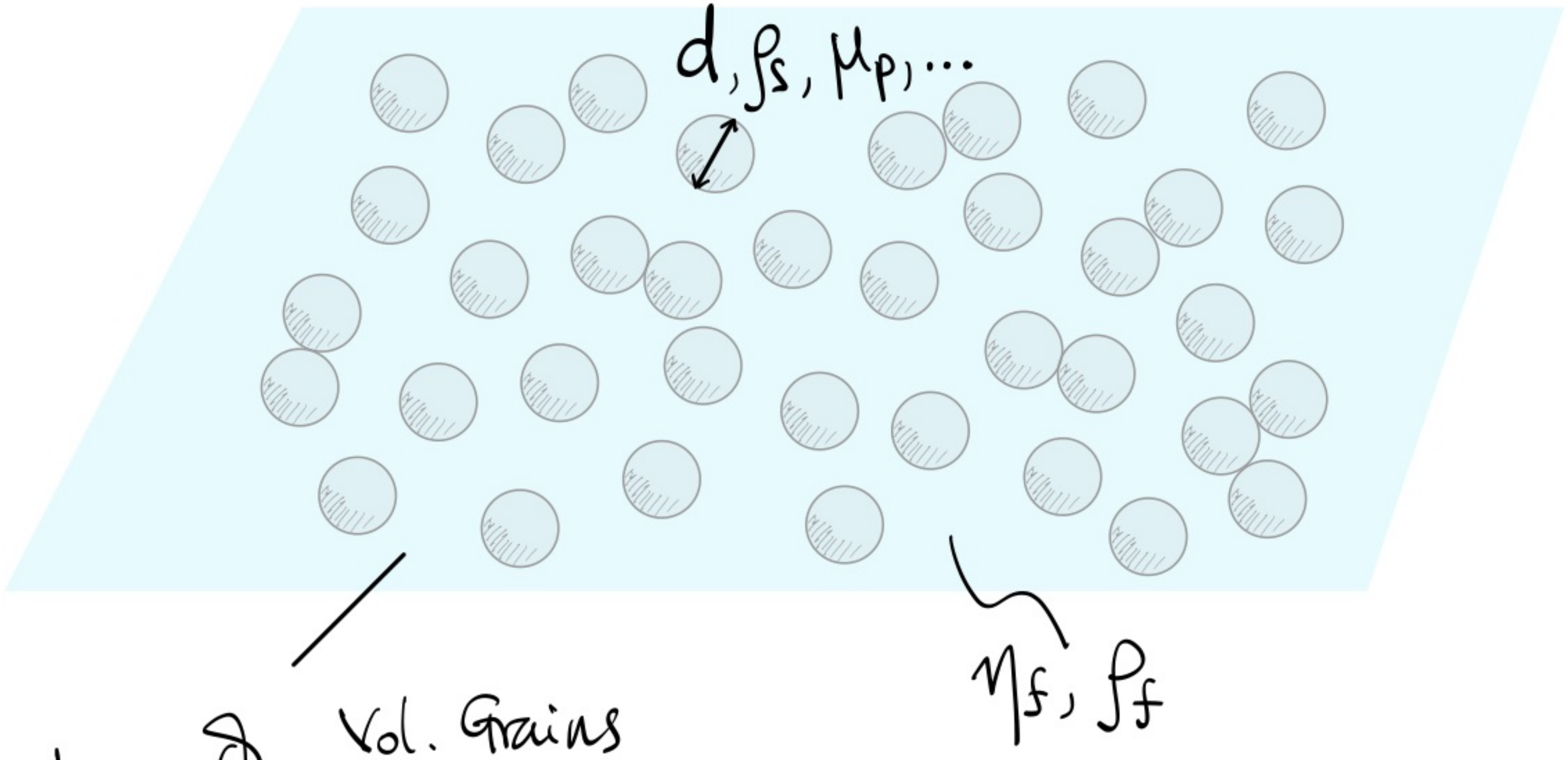
   ii) *The frictional transition model*

   ii) *Hydrodynamics of shear—thickening suspensions*

What is a Suspension?

Definition:

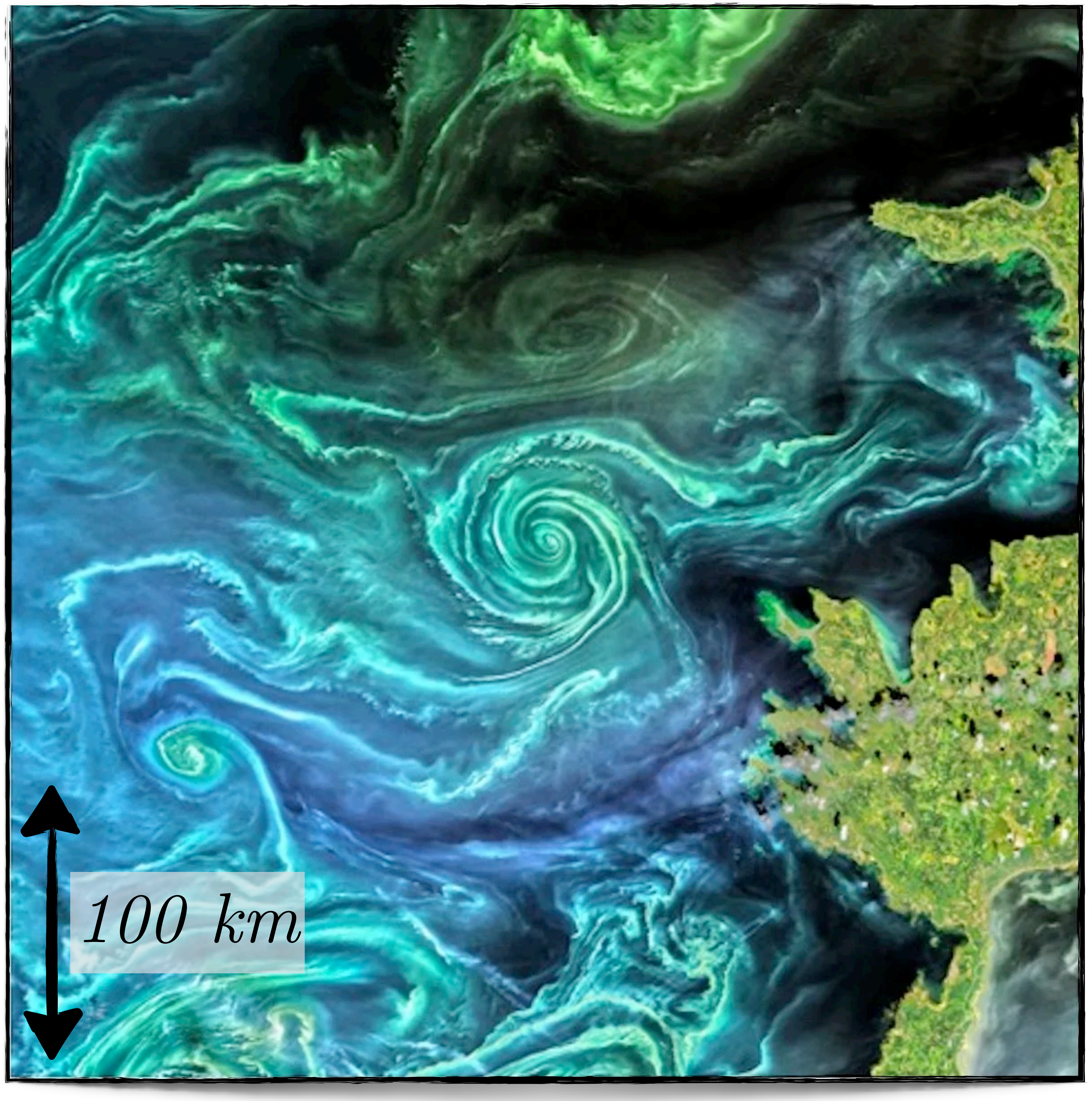
Generally speaking, it's a mixture of solid particles and a fluid (e.g. water, oil ...)



Volume fraction, $\Phi = \frac{\text{Vol. Grains}}{\text{Total Vol.}}$

Suspensions in Nature:

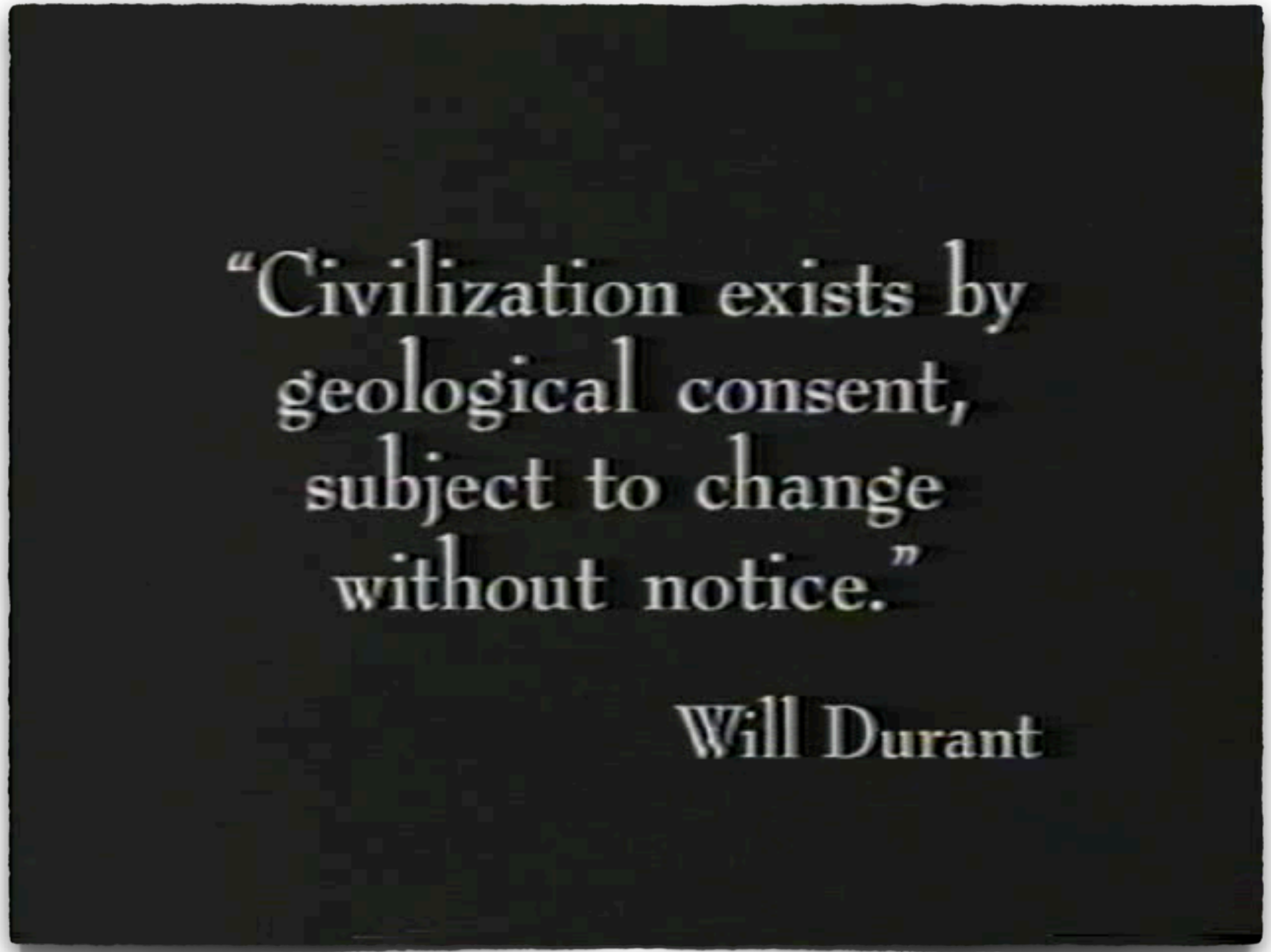
e.g. Phytoplankton blooms in Gulf of Finland



Suspensions in Nature:

e.g. Geophysical Flows

Debris flows, pyroclastic flows, snow avalanches, among others...



"Civilization exists by
geological consent,
subject to change
without notice."

Will Durant

Suspensions in Nature:

e.g. Geophysical Flows

Debris flows, pyroclastic flows, snow avalanches, lahars, among others...

Suspensions in Nature:

e.g. Geophysical Flows

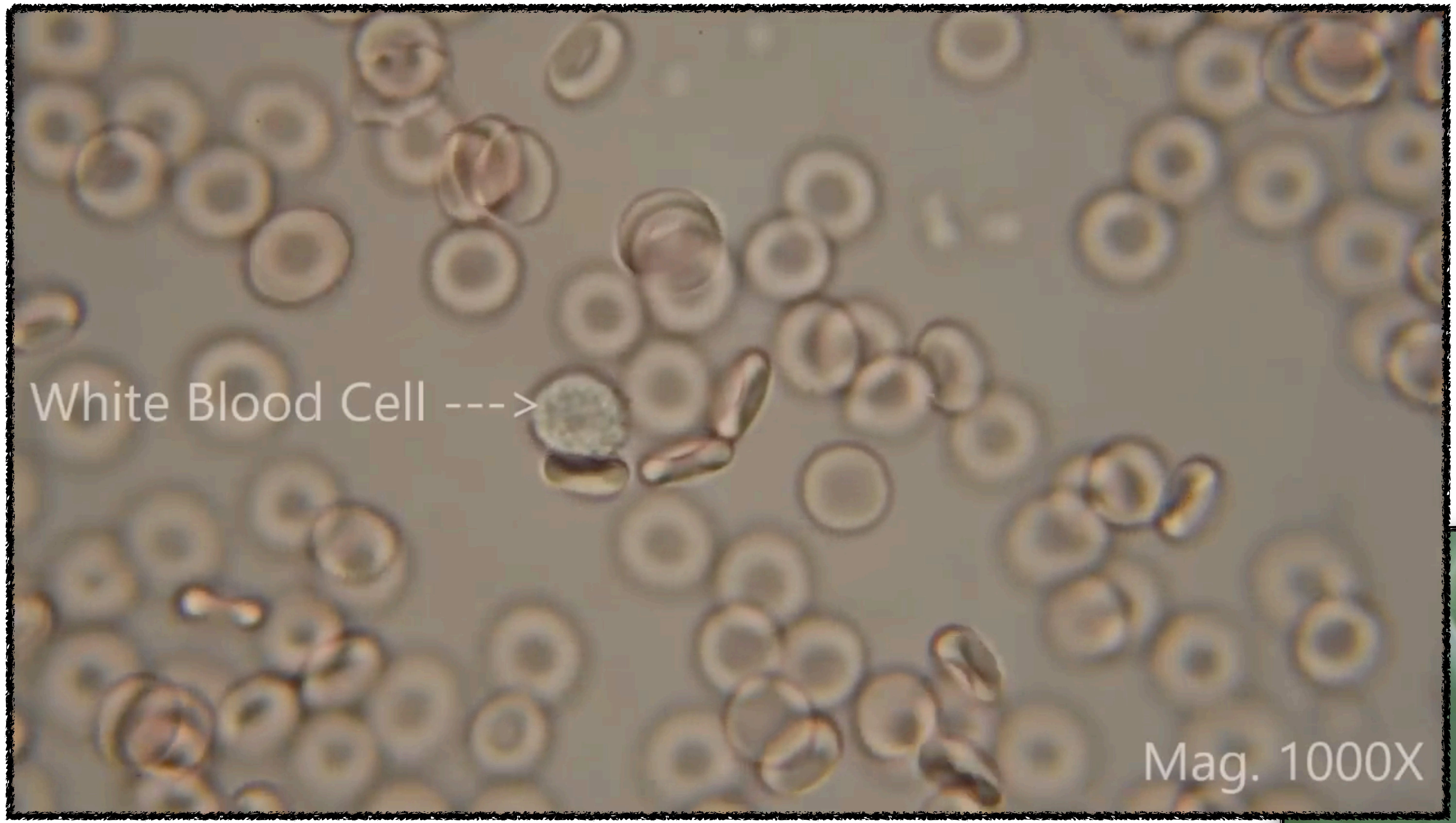
Debris flows, pyroclastic flows, snow avalanches, lahars, among others...

Between 2004 and 2016, almost **5000** debris flows were registered globally killing more than **50.000 people**

(Froude & Petley *Nat. Hazards Earth Syst. Sci.* 2018)

Suspensions in Nature:

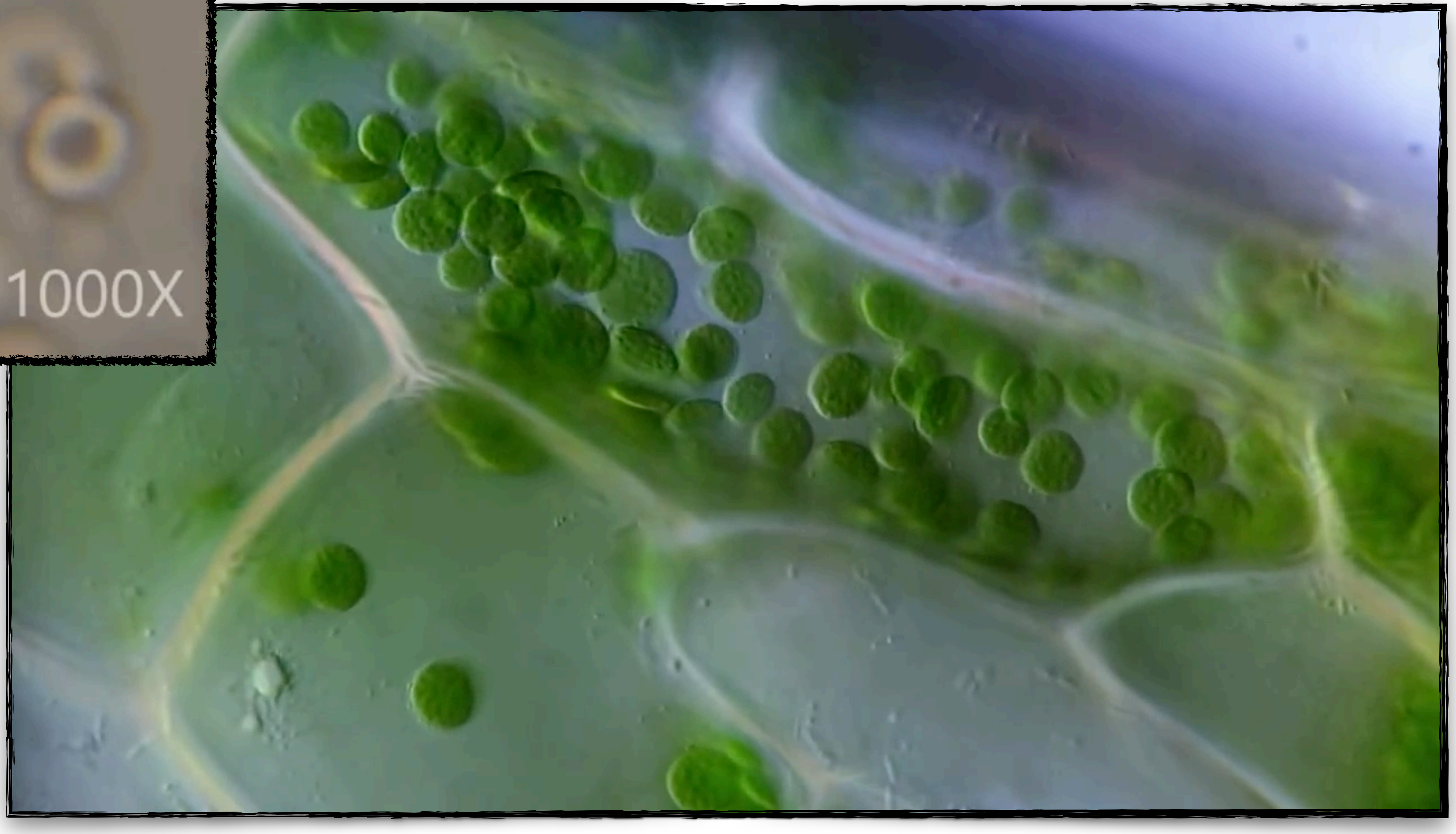
e.g. biological flows



Red blood cells

credits: Sci-Insp Youtube Channel

Cytoplasmic streams



Suspensions in Industry

Civil engineering

Food processing

Paints, bulk chemicals...

Only in the USA, **10^{12} kg** of granular materials are used **per year.**
2nd most—used product!

(Shinbrot & Muzzio PRL 1998)

Not Trivial to Understand Scientifically…

Multidisciplinary Interest:

statistical physics, fluid / solid mechanics, soil mechanics

Not trivial to apply "classical" techniques:

In many situations thermal agitation does not play a role
Extremely dissipative system ☒ *Classical statistical mechanics*

Real situations involves HUGE amount of particles ☒ *Tracking individual grains*

Suspensions as
an effective Complex Fluid

/

Suspension Rheology???

Different Physics Depending on Particle Size

Brownian/colloidal suspensions

Granular suspensions

$1\ \mu m$

$100\ \mu m$

Grain size

d

Thermal agitation

Repulsive forces (Electrostatics, polymer brushes...)

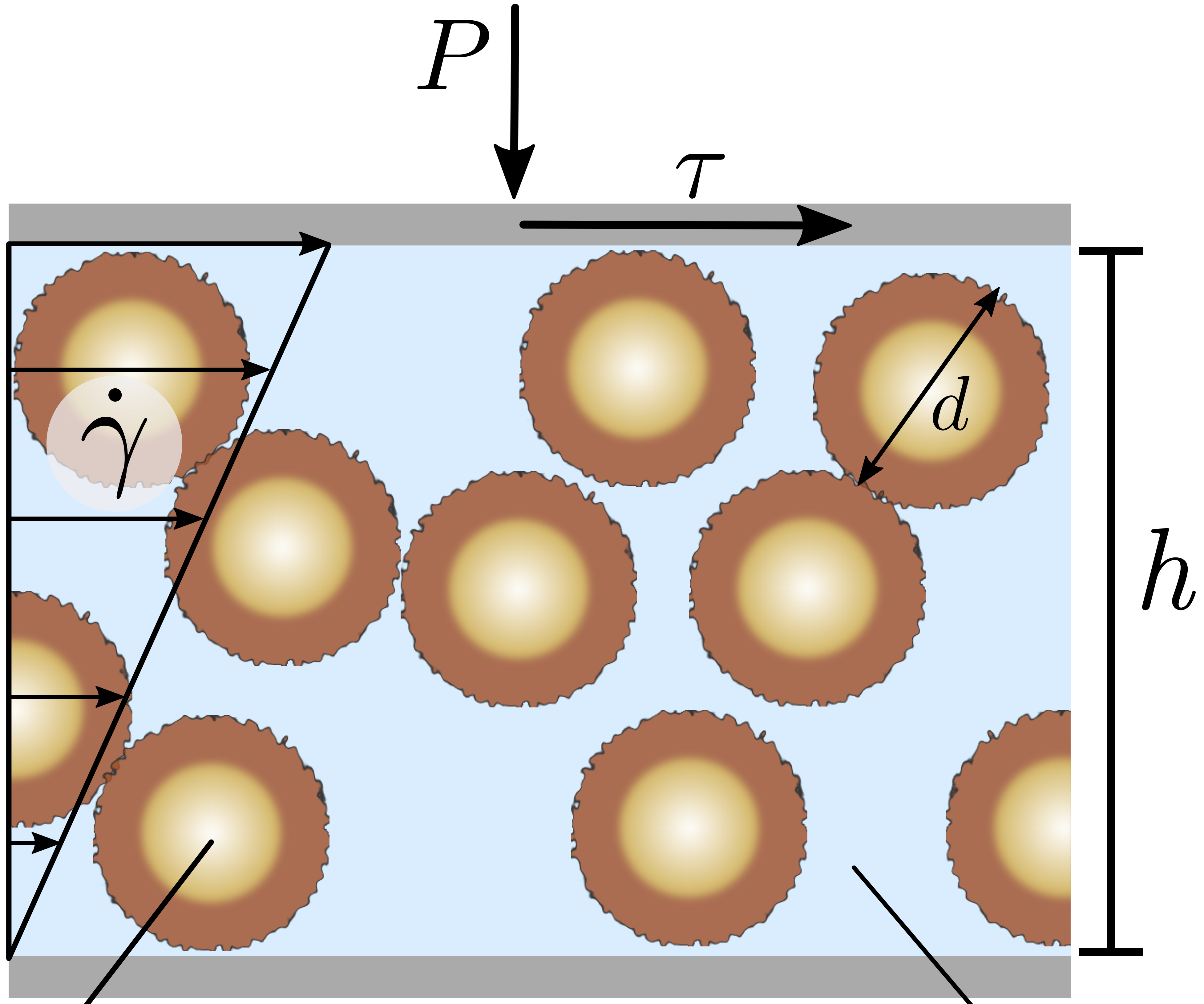
Adhesion forces (Van der Waals,...)

Hydrodynamics

Solid contacts

Friction

Rheology of Granular Suspensions $\left\{ \rho_p = \rho_f \qquad Re = \frac{\rho_f \dot{\gamma} h^2}{\eta_f} \ll 1 \qquad St = \frac{\rho_p \dot{\gamma} d^2}{\eta_f} \ll 1 \right\}$

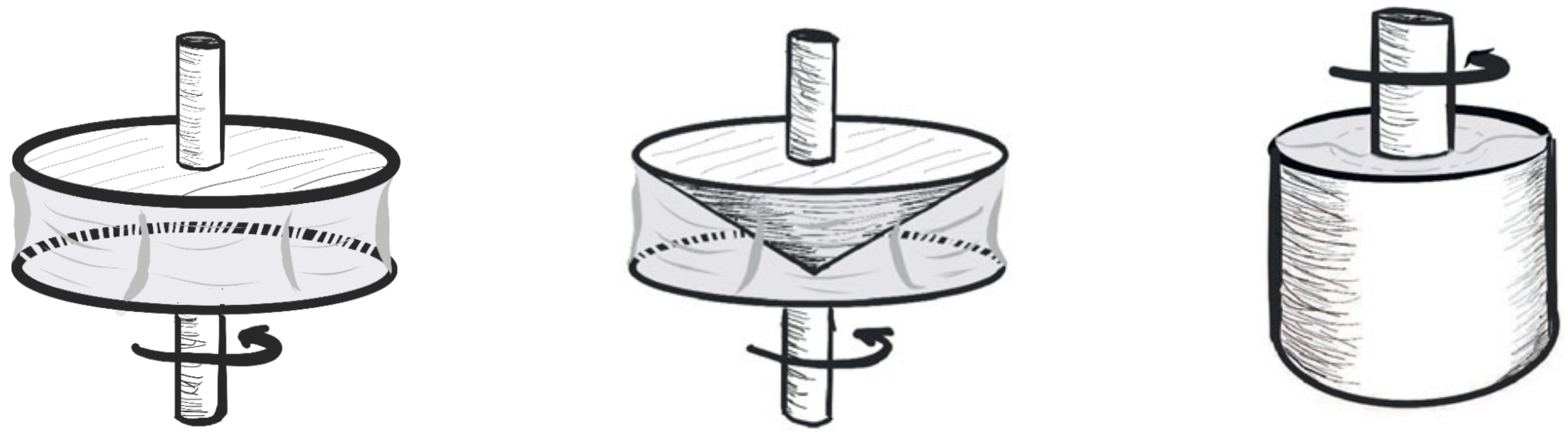


Non-Brownian, rigid
& frictional particles

viscous fluid
η_f , ρ_f

$$\phi = \frac{\text{Vol of Grains}}{\text{Total Volume}}$$

"Classical" Rheology (volume—imposed)



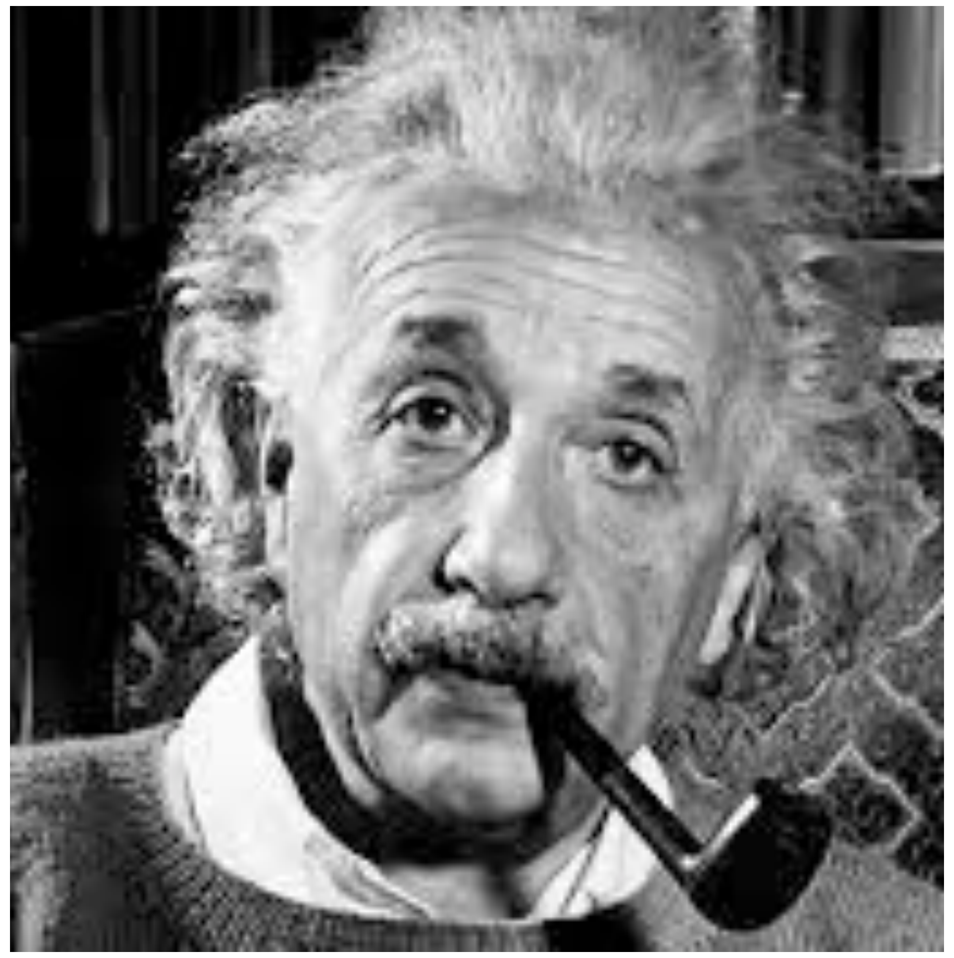
Volume Fraction, ϕ Shear Rate, $\dot{\gamma}$	**Imposed**
Shear Stress , τ	**Measured**

$$\tau = \boxed{\eta_s(\phi)}^{??} \eta_f \dot{\gamma}$$

Suspension Effective Viscosity

Chronology & ϕ

1906



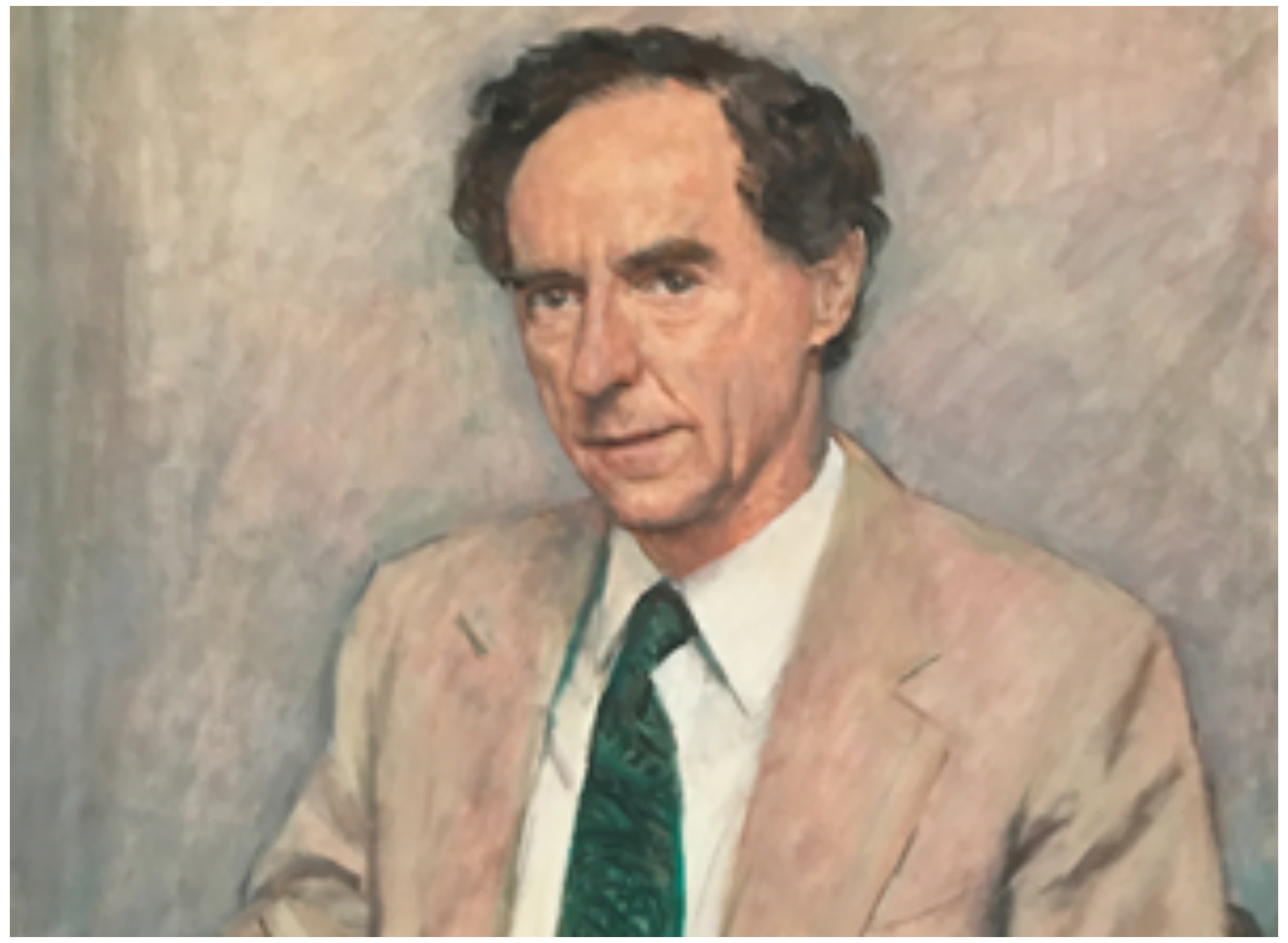
(A. Einstein PhD Thesis 1906)

$$\eta_s(\phi) = 1 + 5\phi/2$$

Stresslet dissipation

— *Dilute regime* —

1972



(Batchelor & Green JFM 1970)

$$\eta_s(\phi) = 1 + 5\phi/2 + \lambda\phi^2$$

Pair interactions

— *Semi-dilute regime* —

2000

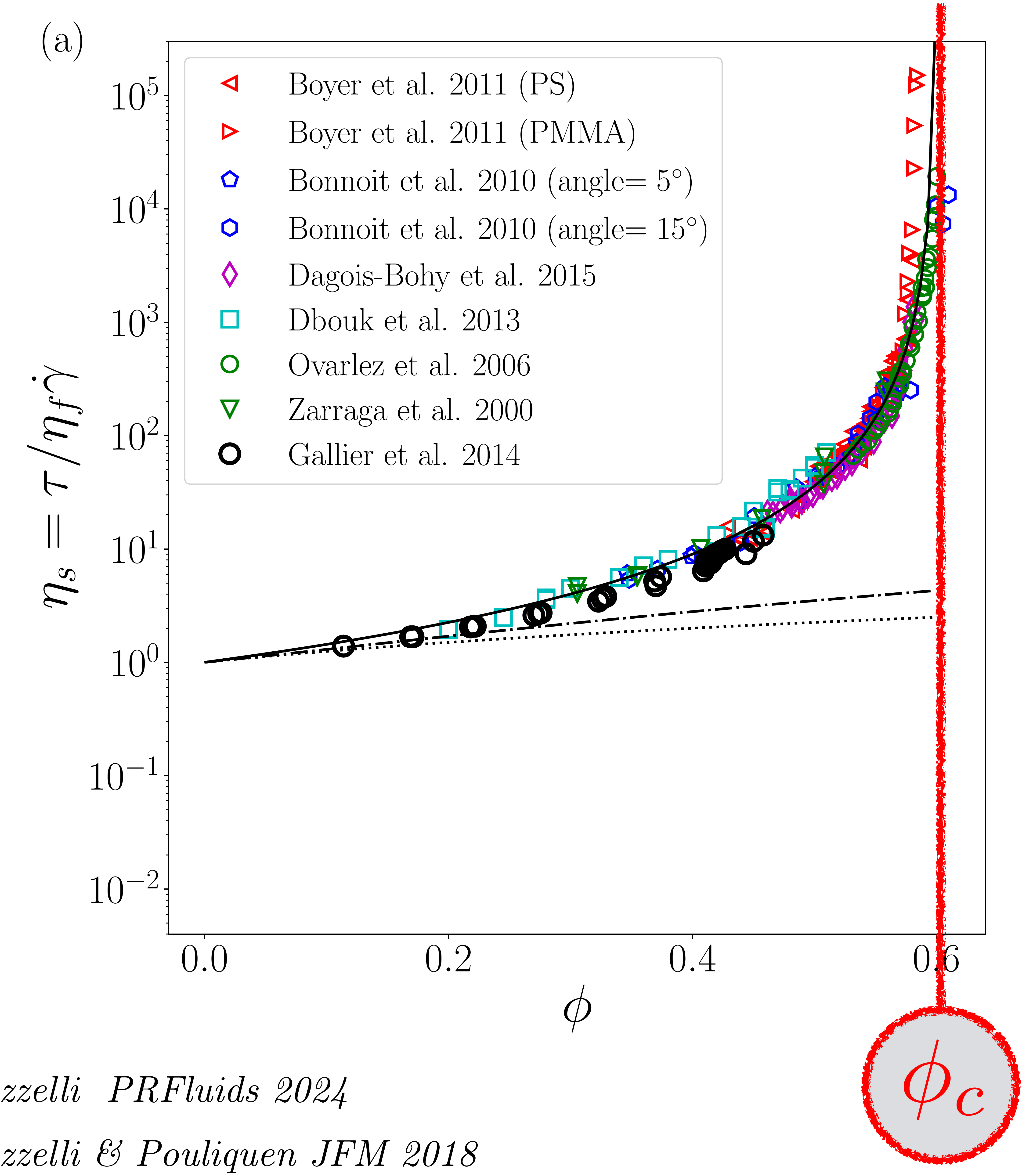
What about dense regime?

Multibody interactions:

no exact analytical solution!

Guazzelli & Pouliquen JFM (2018)

Suspension Effective Viscosity: Empirical Observations



$$\tau = \eta_f \eta_s(\phi)\dot{\gamma}$$

$$\eta_s(\phi) \sim (\phi_c - \phi)^{-2}$$

Viscosity is set by the distance to Jamming!

Rate—independent viscosity!

Guazzelli PRFluids 2024

Guazzelli & Pouliquen JFM 2018

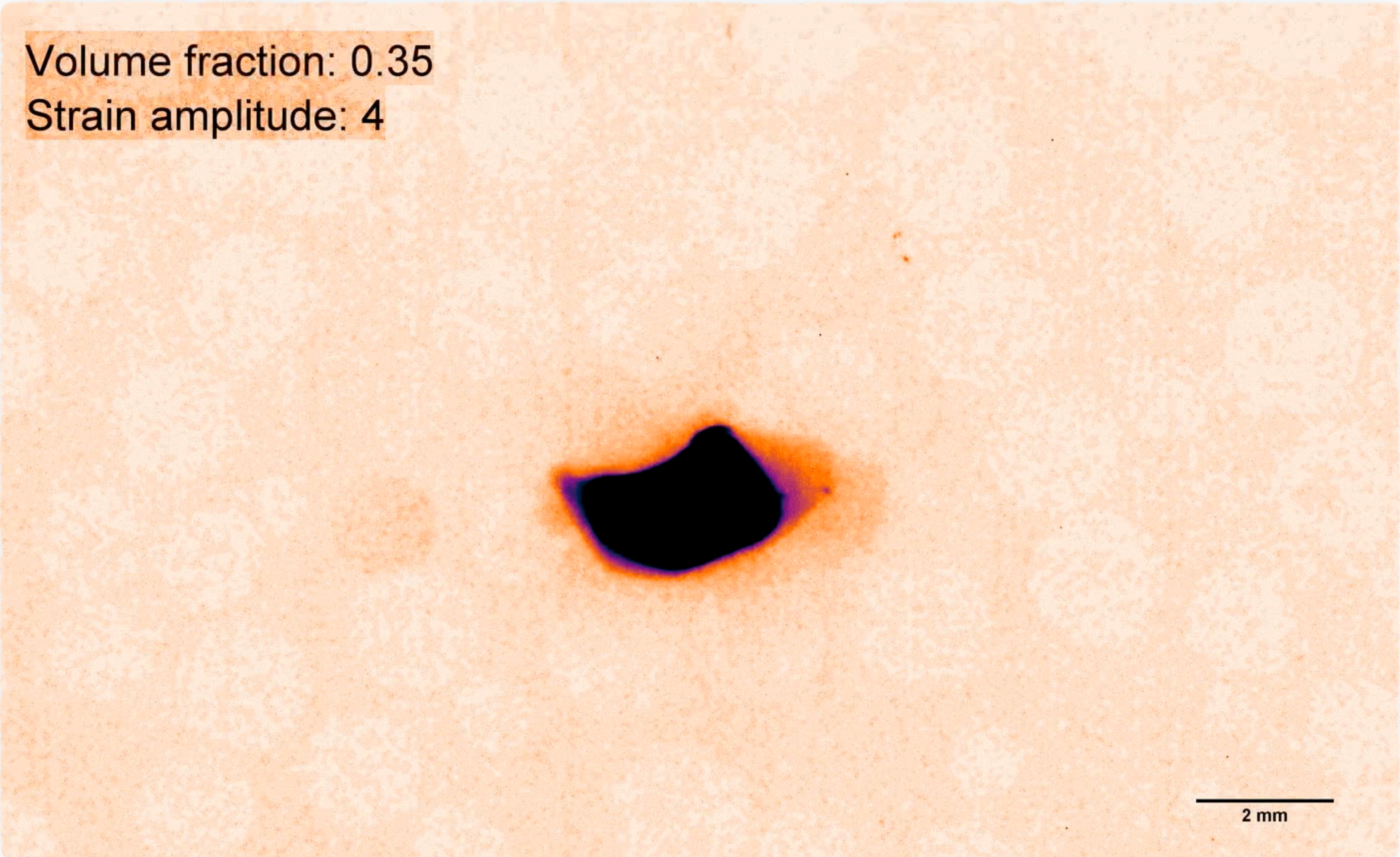
Even the simplest case introduces strange behaviours...

e.g. loss of reversibility

Taylor's experiment with pure viscous fluid

G.I. Taylor (1967)

Taylor's experiment in a granular suspension

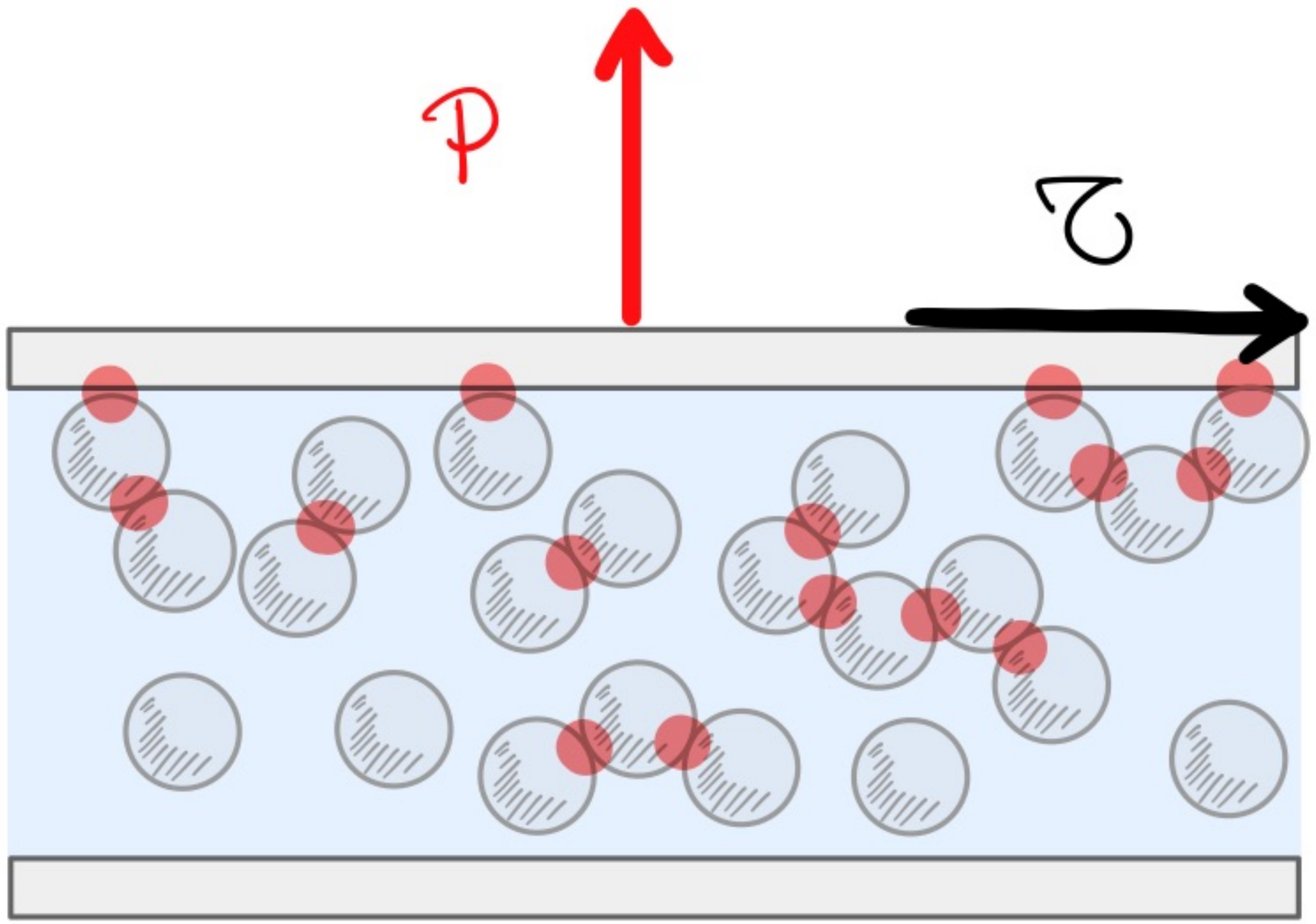


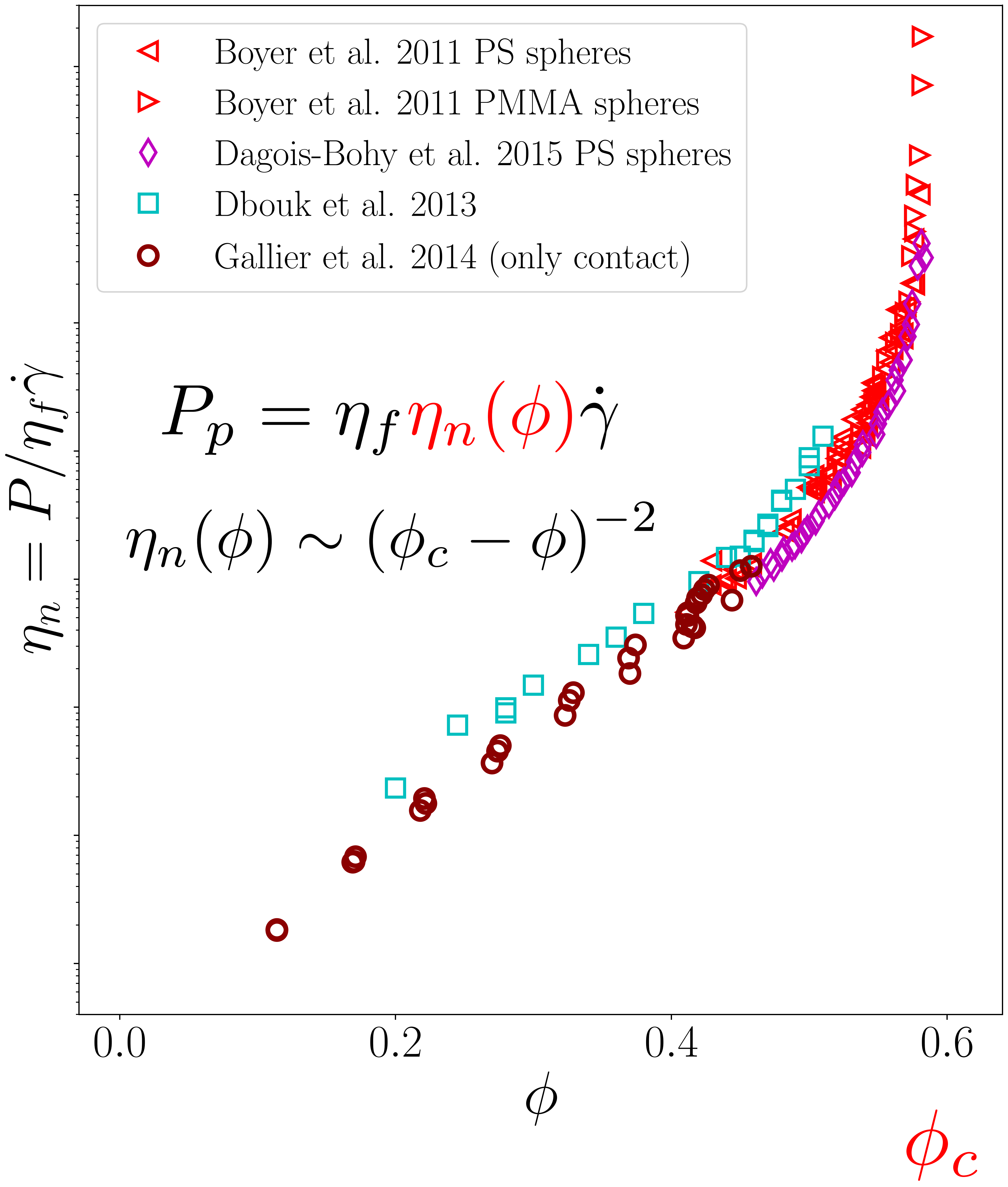
Souzy, Phom & Metzger PRF (2016)

Pine et al. Nature (2013)

Even the simplest case introduces strange behaviours...

e.g. Particle Pressure





$$P_p = \eta_f \eta_n(\phi)\dot{\gamma}$$

$$\eta_n(\phi) \sim (\phi_c - \phi)^{-2}$$

Guazzelli PRFluids (2024)

Guazzelli & Pouliquen JFM (2018)

Dominant Interactions

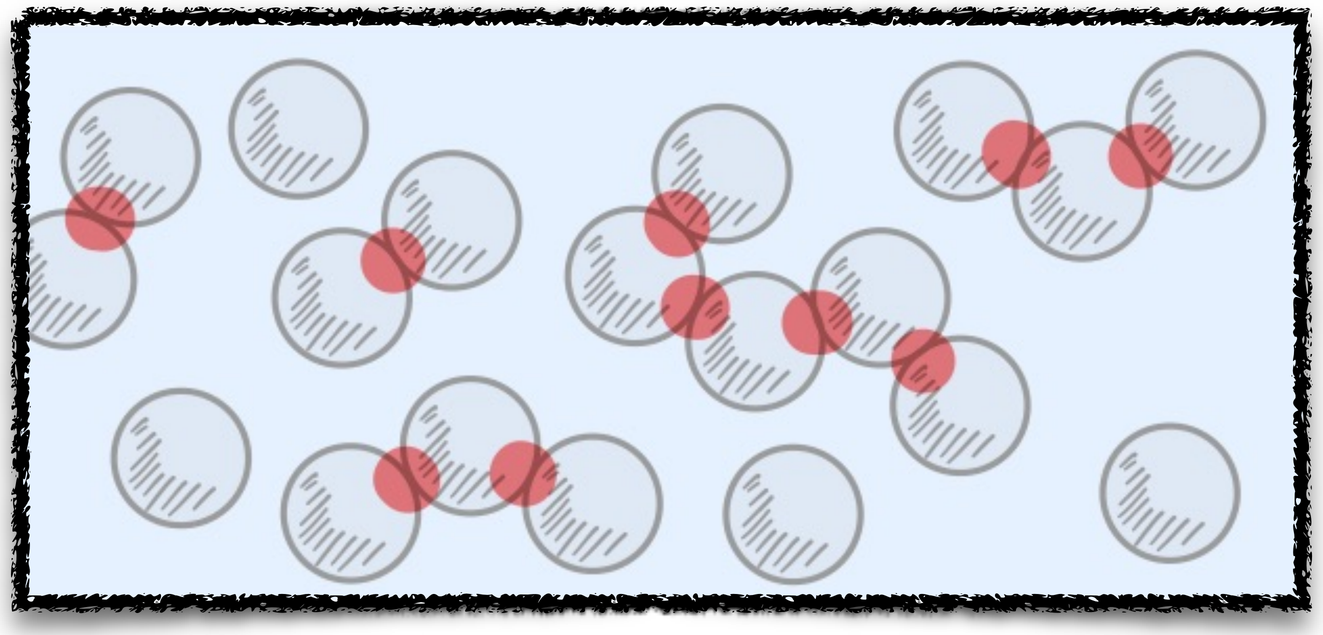
Guazzelli & Pouliquen JFM (2018)

Gallier, Lemaire, Peters & Lobry JFM (2013)

Dilute Regime

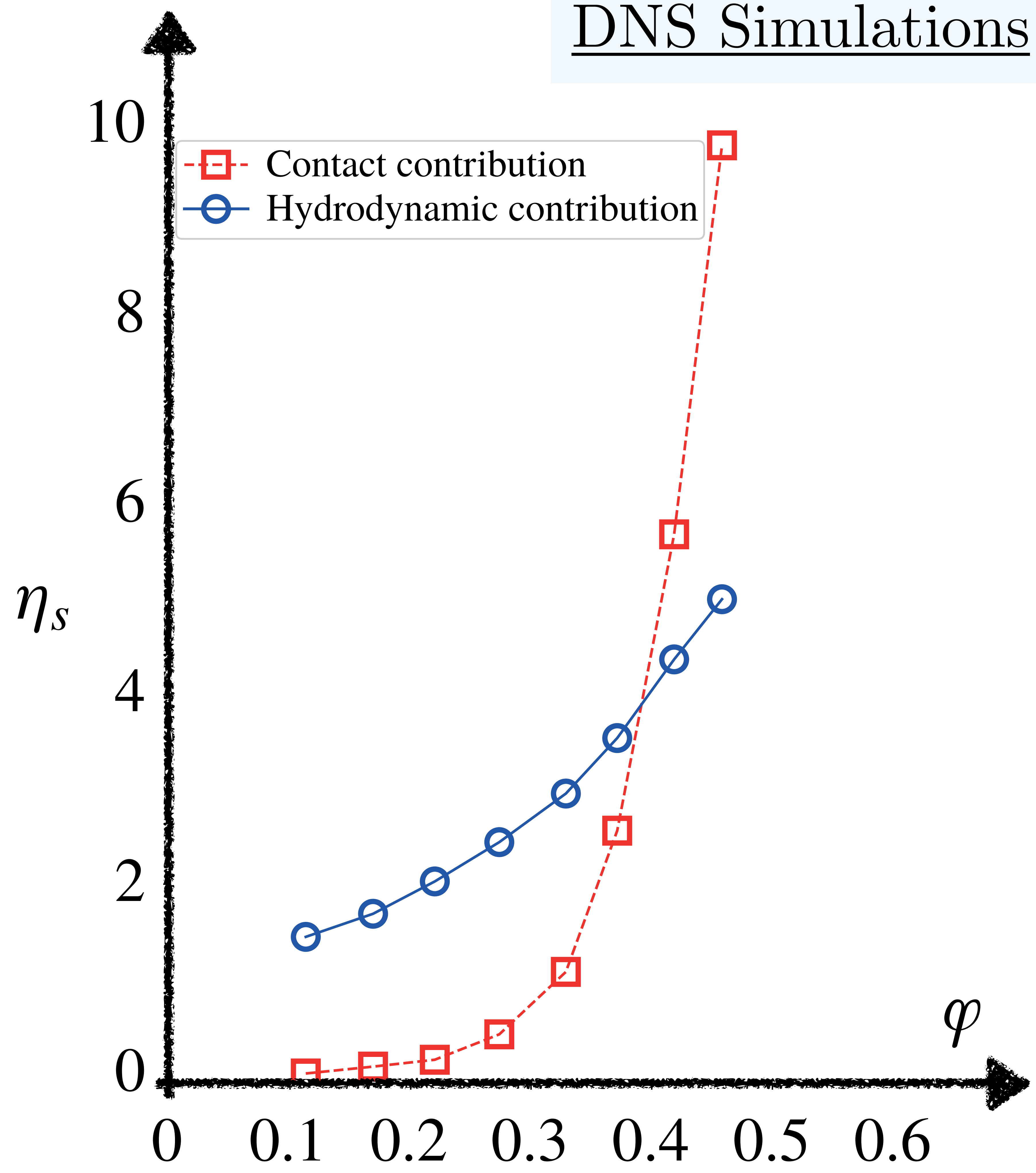
dominated by
long-range hydrodynamic
interactions

Dense Regime



contact dynamics
take over!

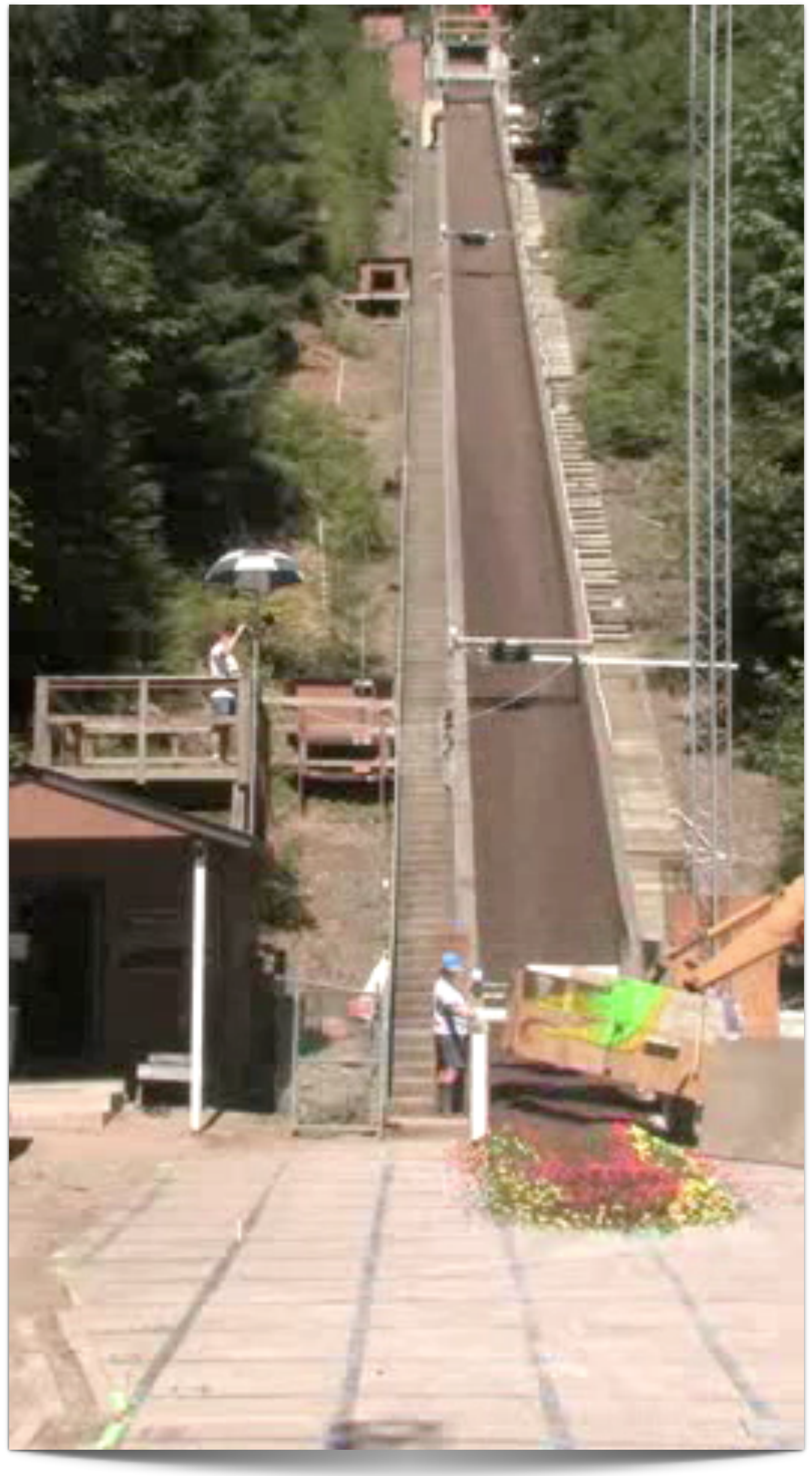
DNS Simulations



An Alternative Approach

GdR MiDi (2004)
da Cruz et al (2005)
Jop, Forterre & Pouliquen (2006)

In many practical situations the volume is not imposed! ⟶ Pressure is imposed (e.g. by gravity!)

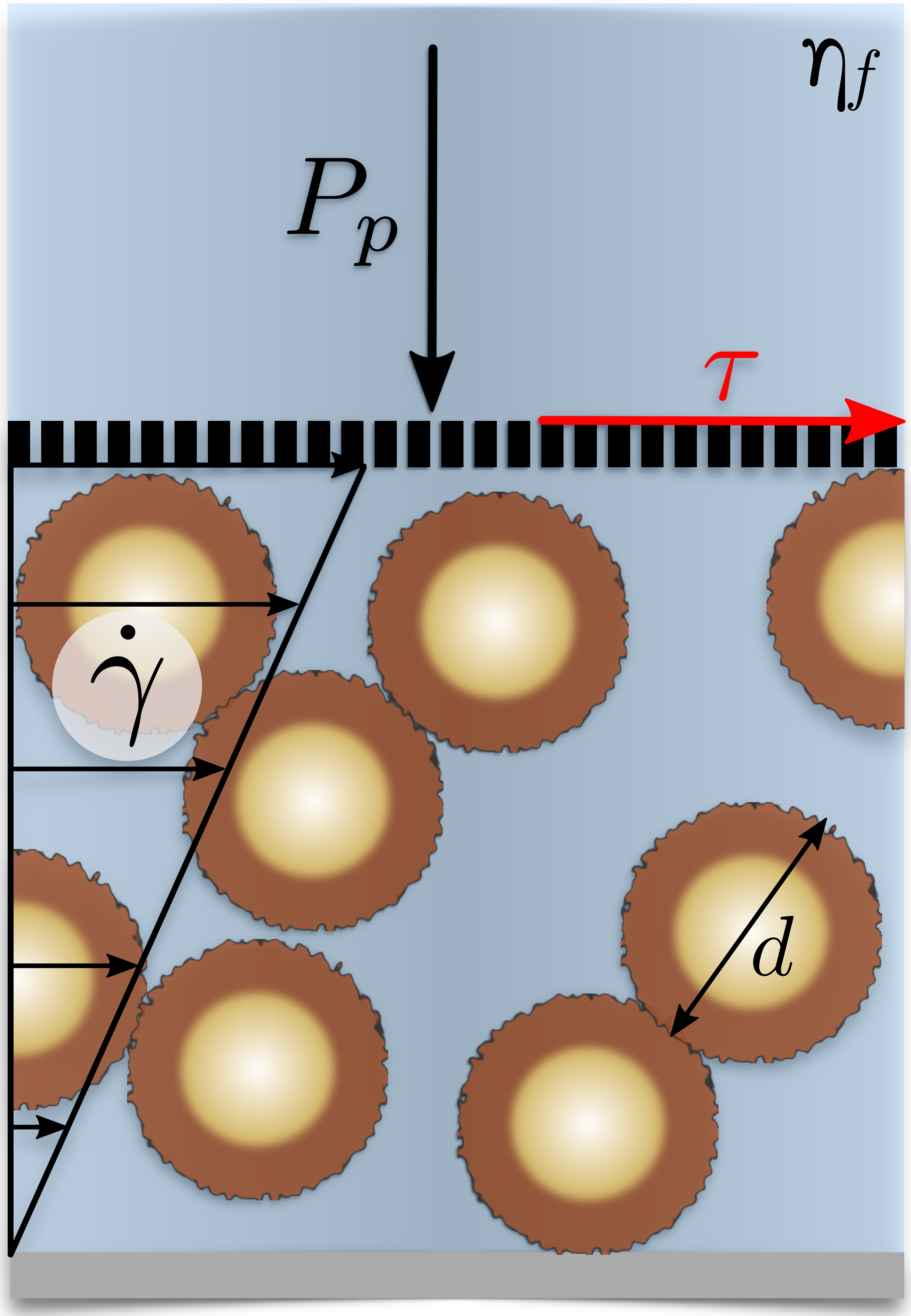


Johnson et al. JGR (2012)

Johnson & Gray JFM (2012)

Rocha et al. JFM (2019)

Pressure—imposed Rheology



Particle Pressure, P_p
Shear Rate, $\dot{\gamma}$ ⟶ Imposed

Shear Stress , τ
Volume Fraction, ϕ Measured

1 dimensionless Number

$$\longrightarrow \quad J = \frac{\eta_f \dot{\gamma}}{P_p}$$

Viscous Number

Viscous stress / Imposed Pressure

Dimensionless Shear Rate

Frictional Rheology:

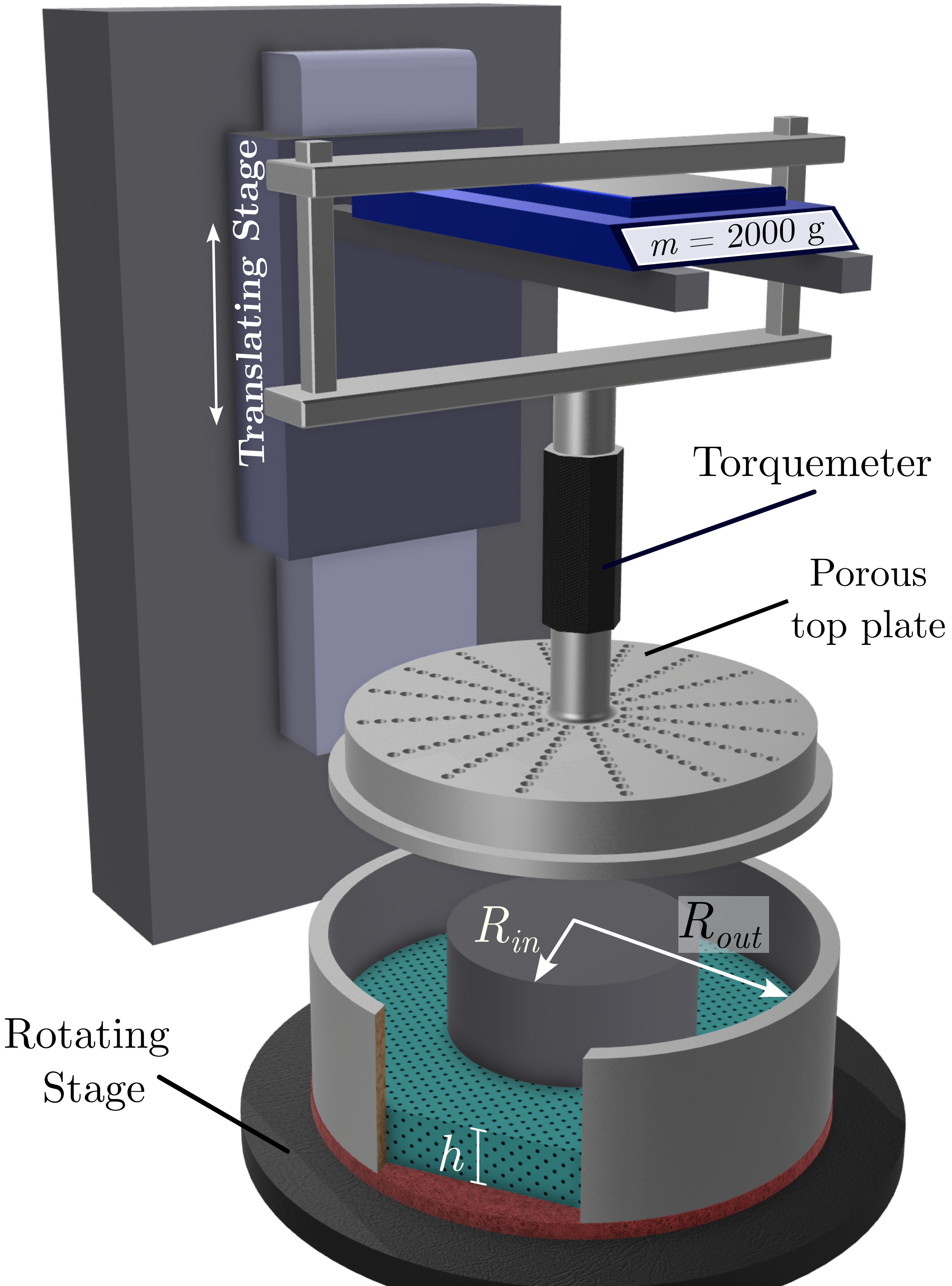
$$\boxed{\begin{aligned} \tau &= \mu(J) P_p \\ \phi &= \phi(J) \end{aligned}}$$

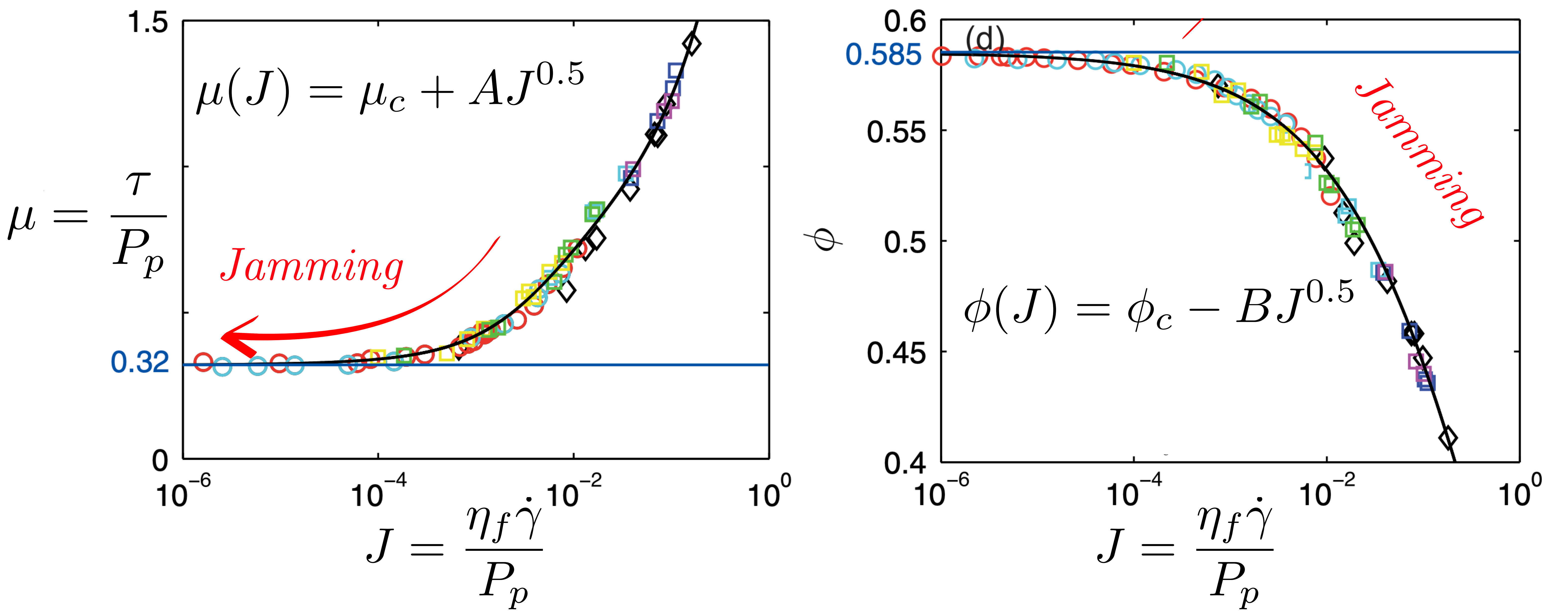
Macroscopic (bulk)
Friction Coefficient

Pressure—imposed Rheology

Boyer, Guazzelli, Pouliquen PRL (2011)

Dagois-Bohy et al. JFM (2015)



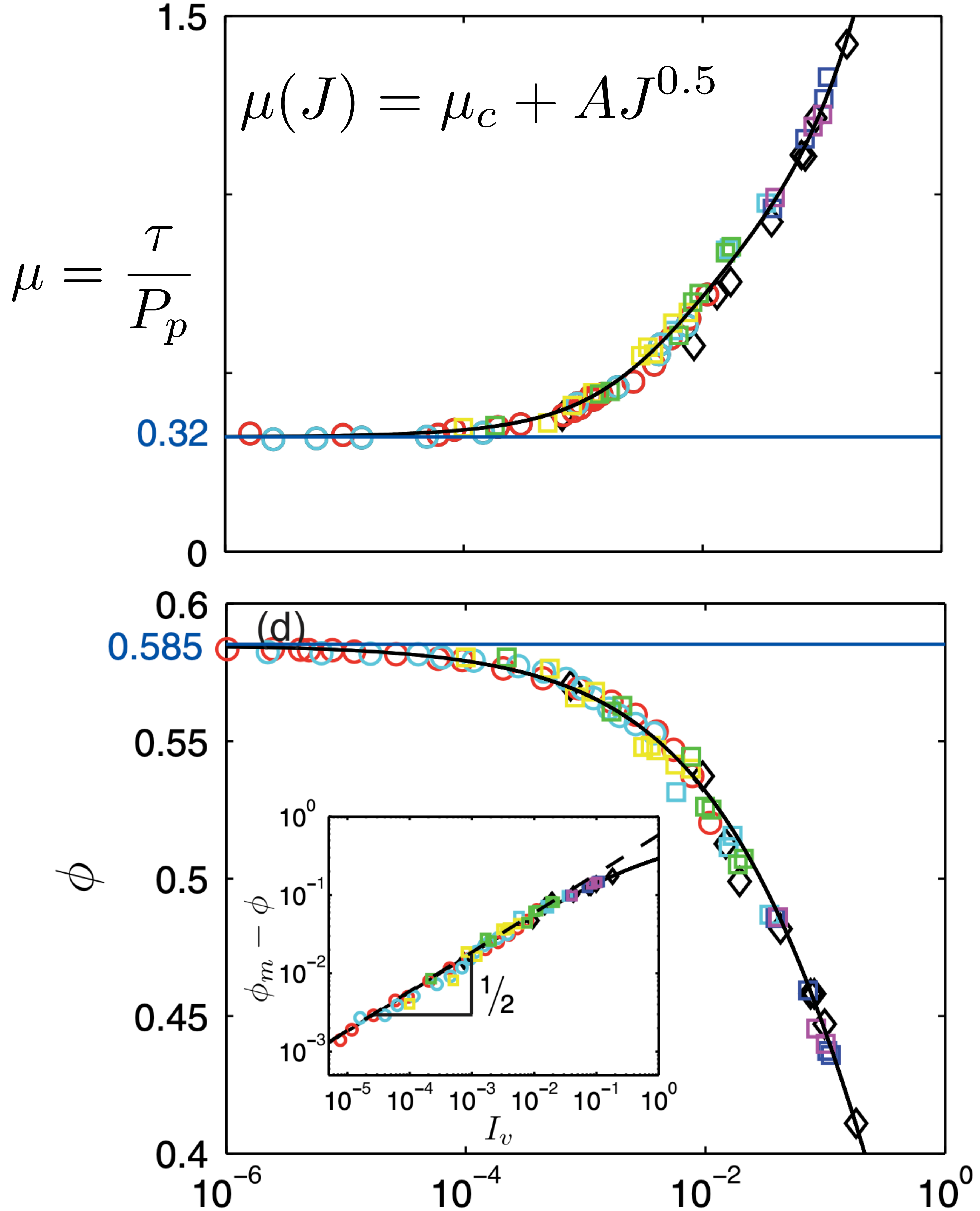


→ *Allows particle pressure P_p to be measured directly.*

→ *Since there is no divergence, it allows to get much closer to jamming!*

Not very accurate in the dilute regime

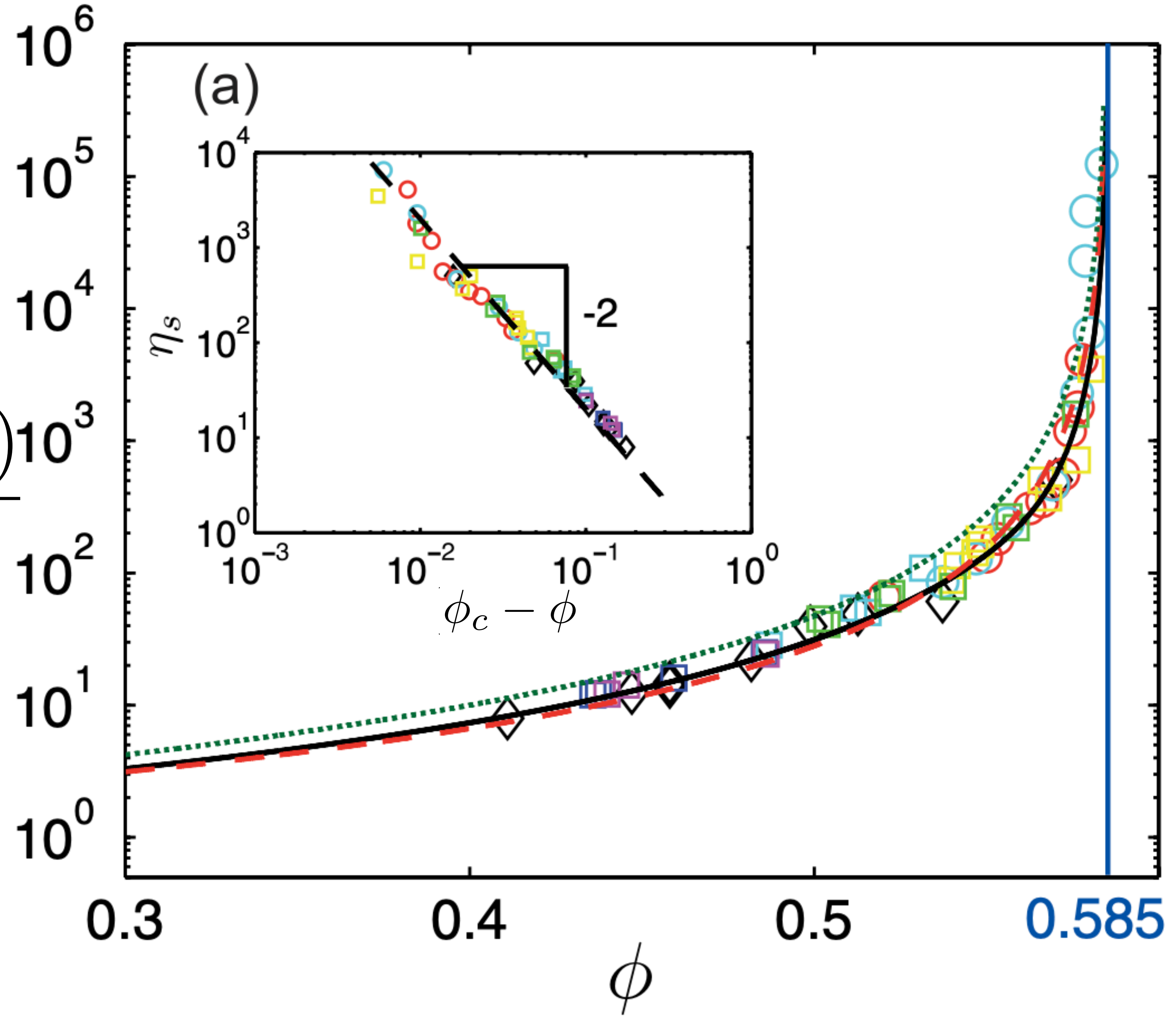
From Pressure— to Volume—Imposed Rheology



$$\mu = \frac{\tau}{P_p}$$

$$J = \frac{\eta_f \dot{\gamma}}{P_p}$$

$$\eta_s = \frac{\mu(J(\phi))}{J}$$



Boyer, Guazzelli, Pouliquen PRL (2011)

So Far...

In the dense regime:

i) Long-range hydrodynamics become weak

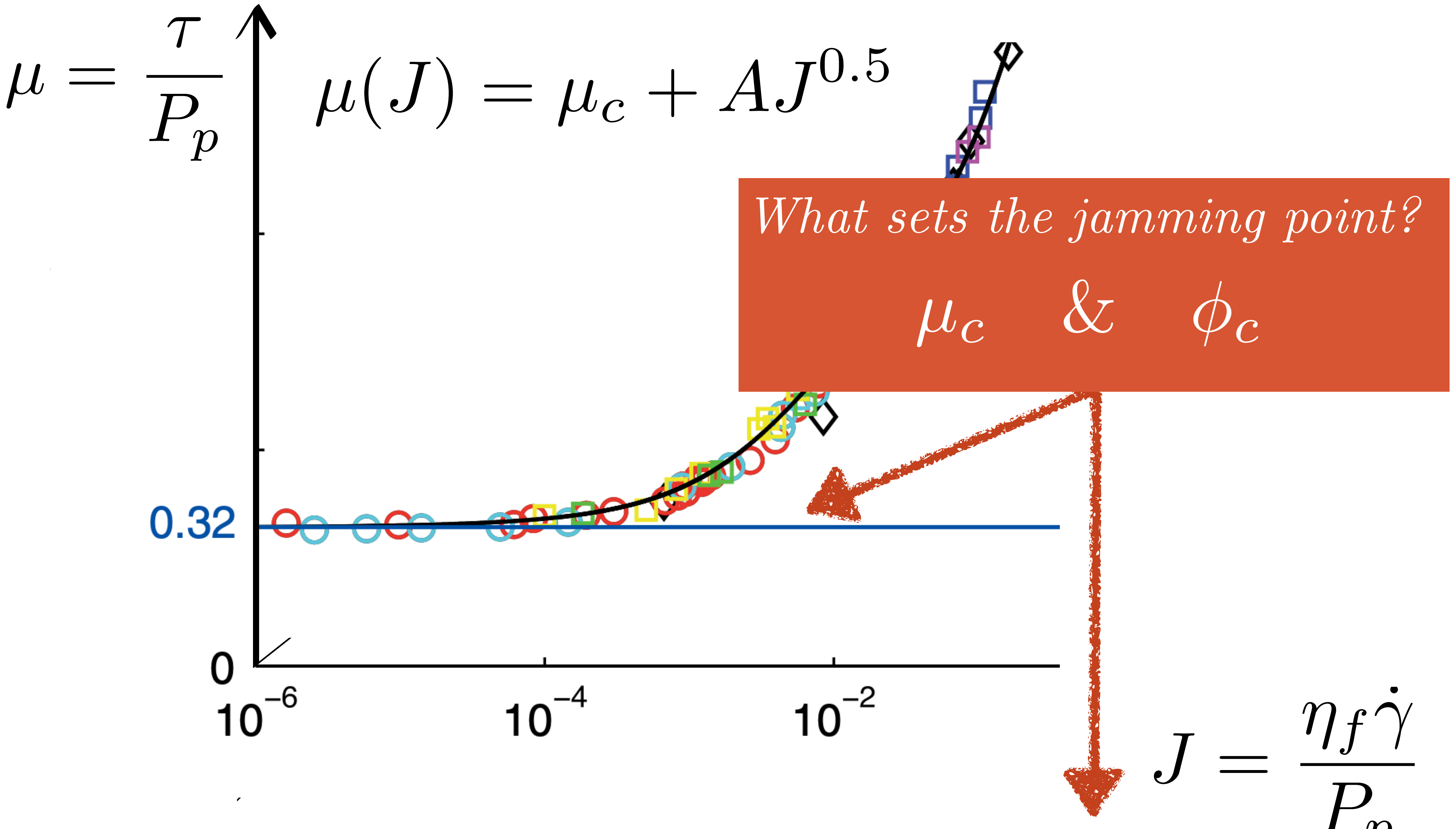
ii) Dynamics is dominated by solid contacts

Volume—imposed *vs* Pressure—Imposed

$$\tau = \eta_f \eta_s(\phi)\dot{\gamma} \qquad \tau = \mu(J) P_p$$

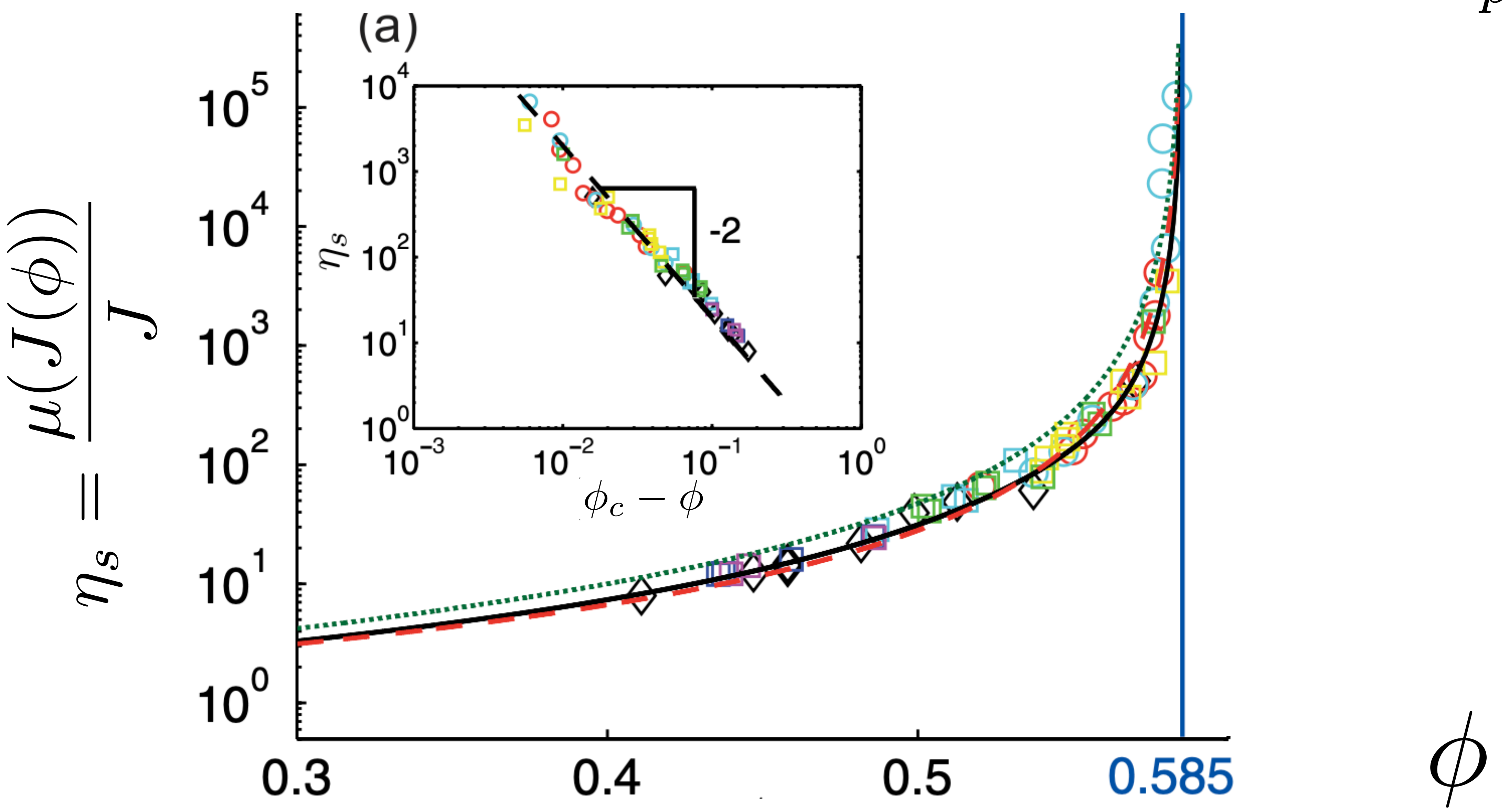
$$P_p = \eta_f \eta_n(\phi)\dot{\gamma} \qquad \phi = \phi(J)$$

Equivalent Flow Rules!



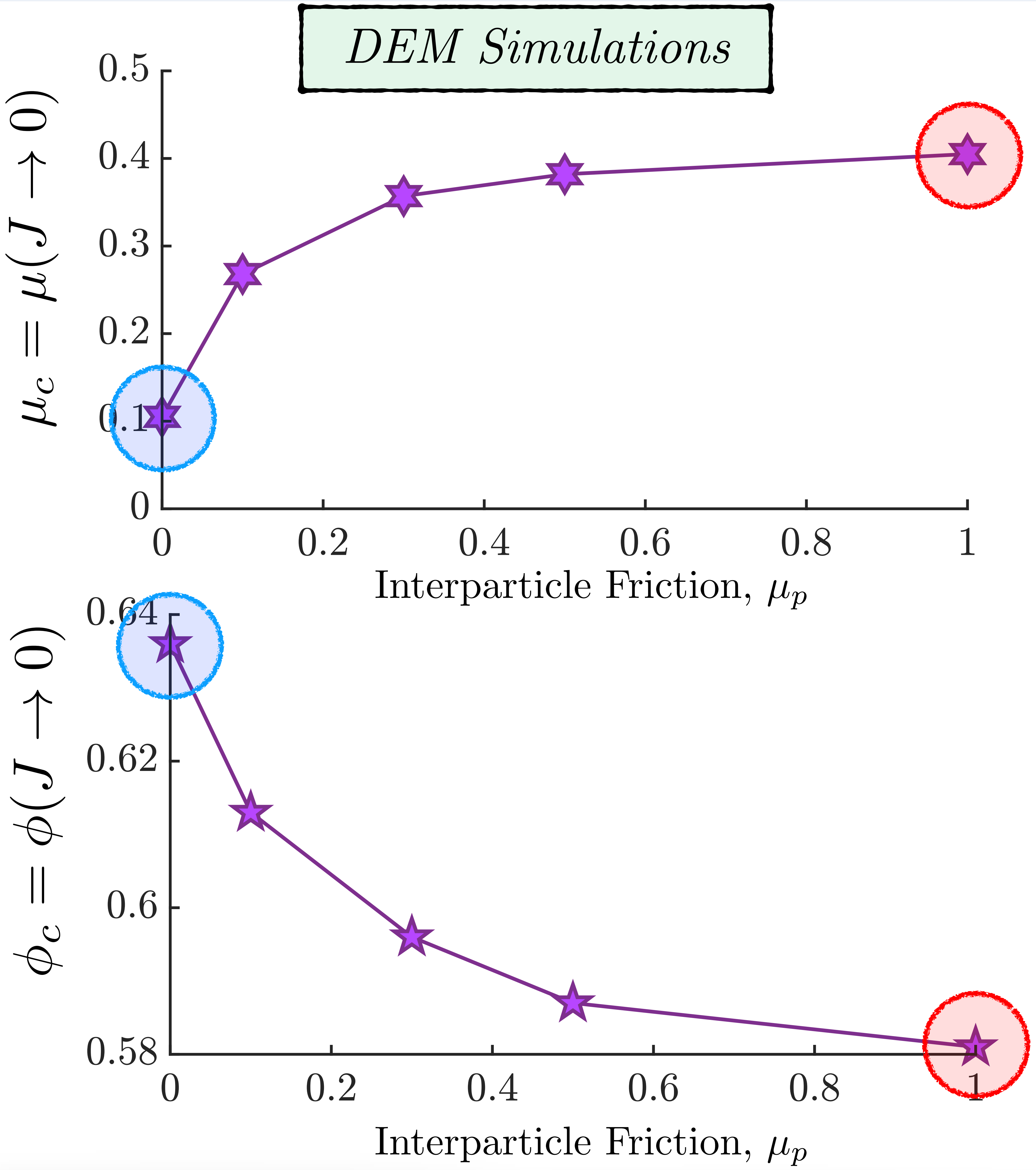
What sets the jamming point?

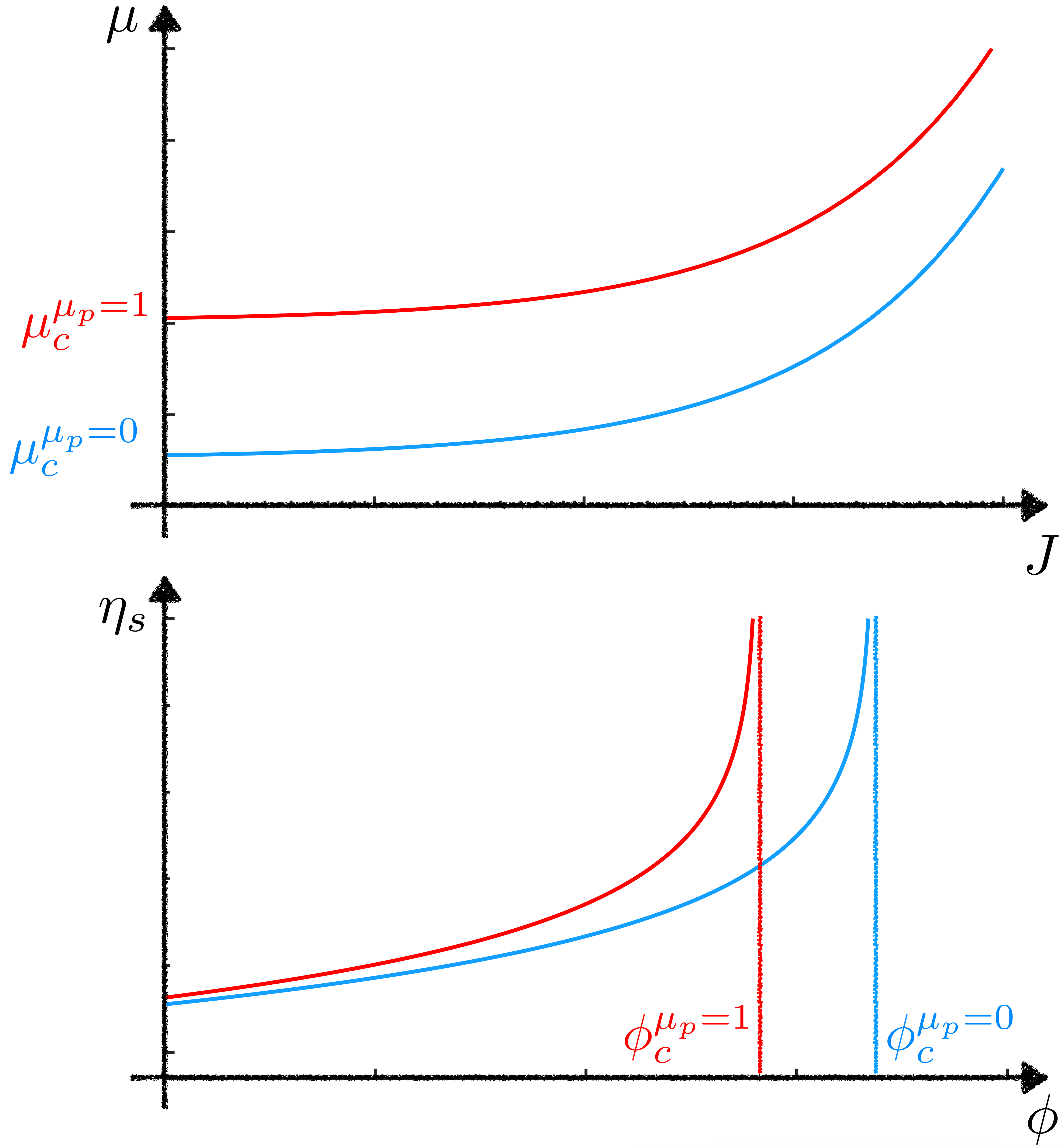
μ_c & ϕ_c



What sets the jamming point?

Chialvo, Sun & Sundaresan PRE (2012)





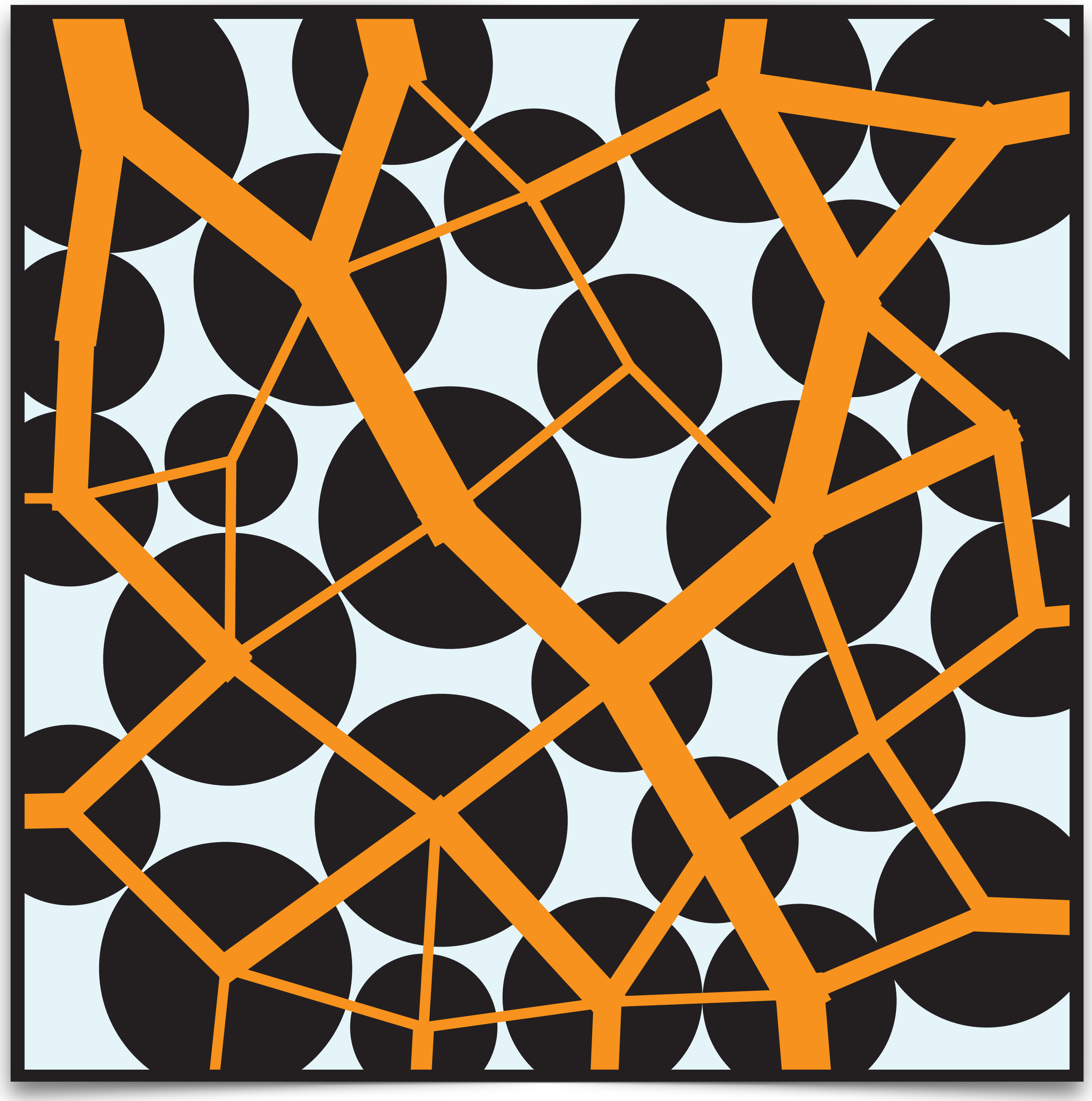
The jamming point depends on the interparticle friction coefficient!!

Why μ_p affects the Jamming Point?

James C. Maxwell

What is the minimum number of contacts to reach rigidity?

Isostaticity — enough contacts to constraint all degrees of freedom



Maxwell's Stability Criterion

Number of Constraints $\geq$ *Degrees of Freedom*

Why μ_p affects the Jamming Point?

Maxwell's Stability Criterion

Number of Constraints $\geq$ *Degrees of Freedom*

Frictionless Contacts:

$$\frac{z_c N}{2} = 3N$$

1 constraint [no penetration]

3 DoF [translation]

$$z_c = 6$$

contacts per particle

Frictional Contacts:

$$\frac{3 z_c N}{2} = 6N$$

3 constraint [1 normal + 2 tangential]

6 DoF [translation + rotation]

$$z_c = 4$$

contacts per particle

What if the contact can switch depending on flow properties such as *stress* or *shear rate*?

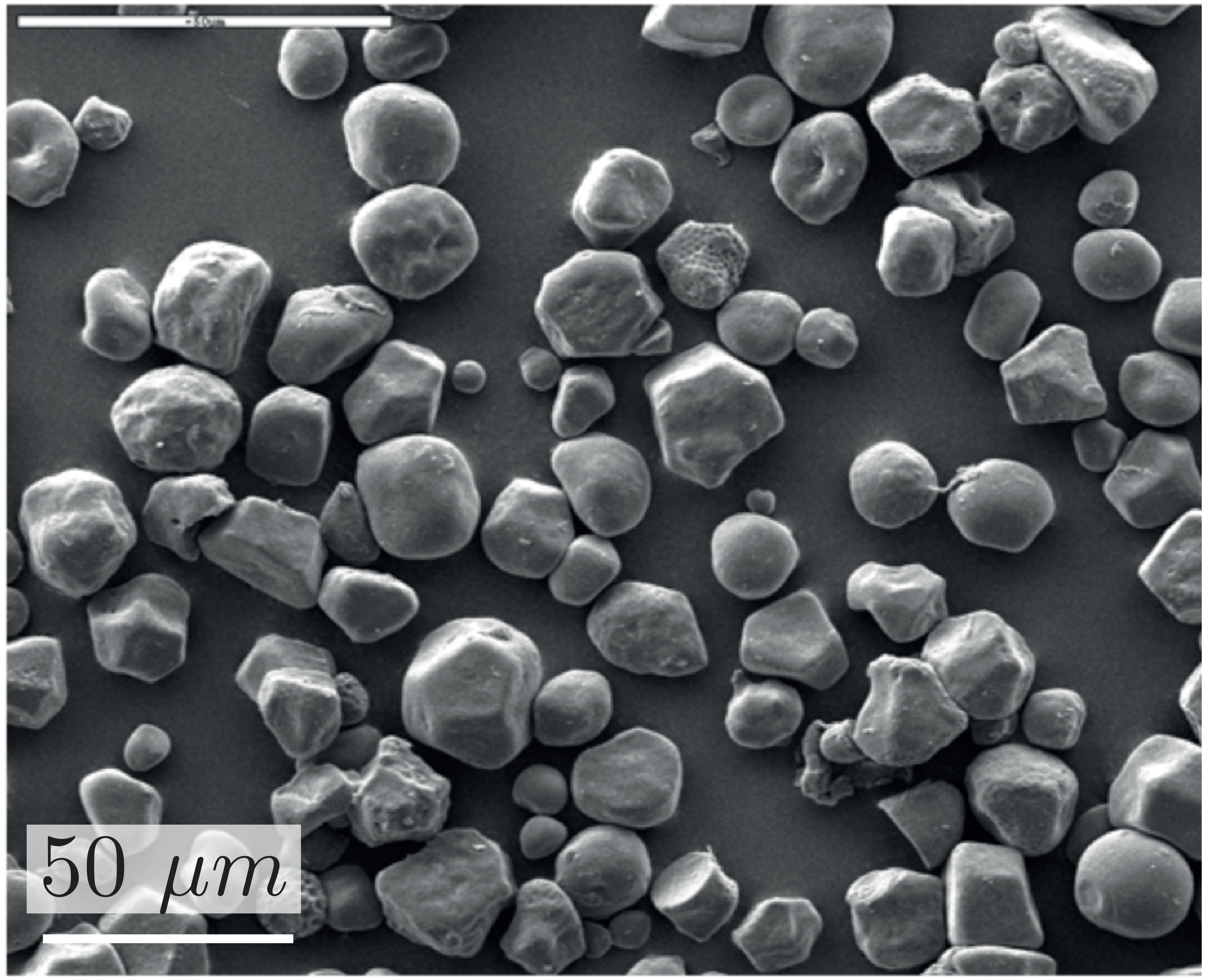
More Recently: Non—Newtonian Behaviours

Hydrodynamics
Solid contacts
Friction

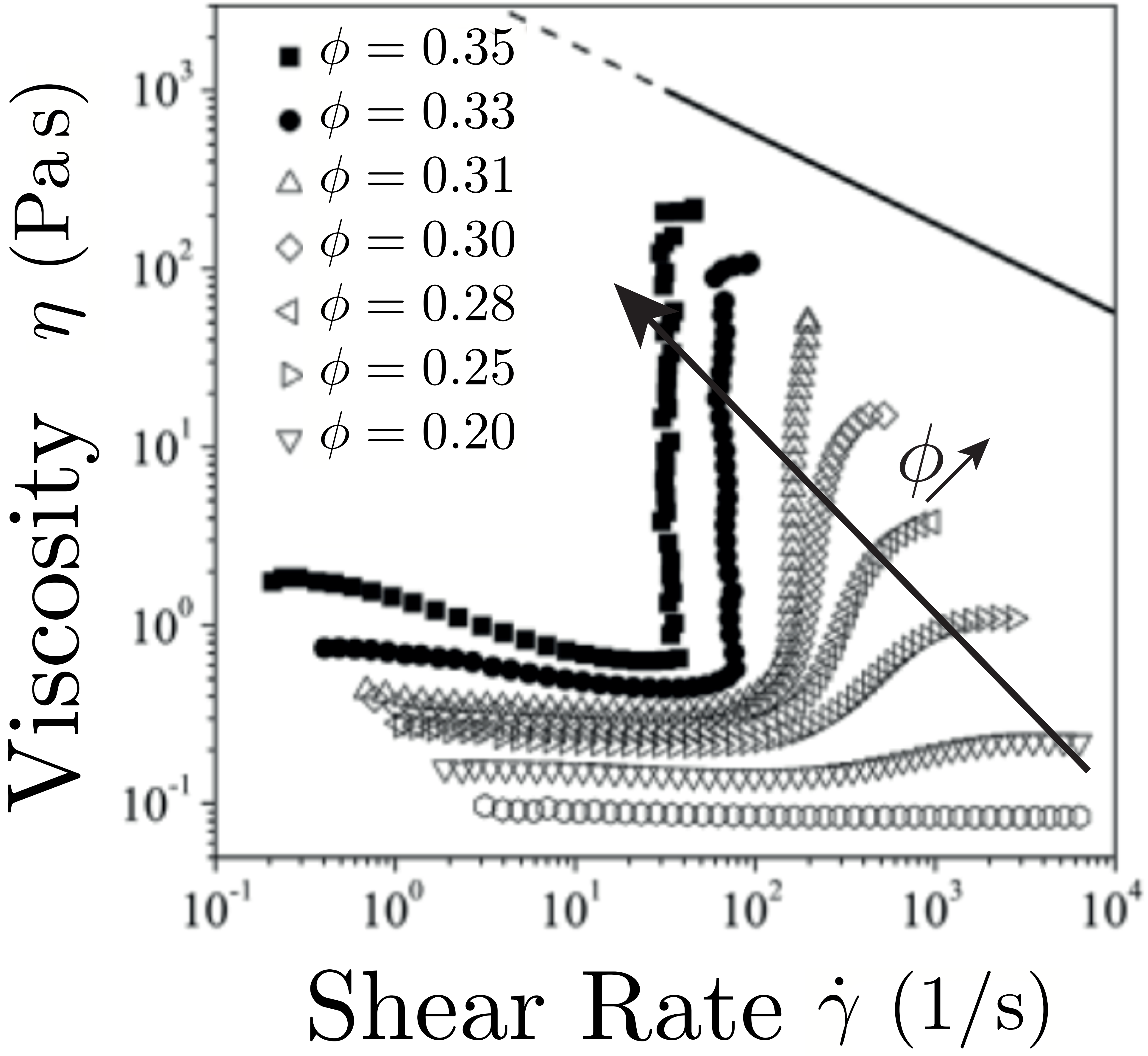
Repulsive forces (Electrostatics, Polymer brushes...)

Adhesion forces (Van der Waals,...)

Shear—Thickening Suspensions: What are they?



e.g. Cornstarch particles in water

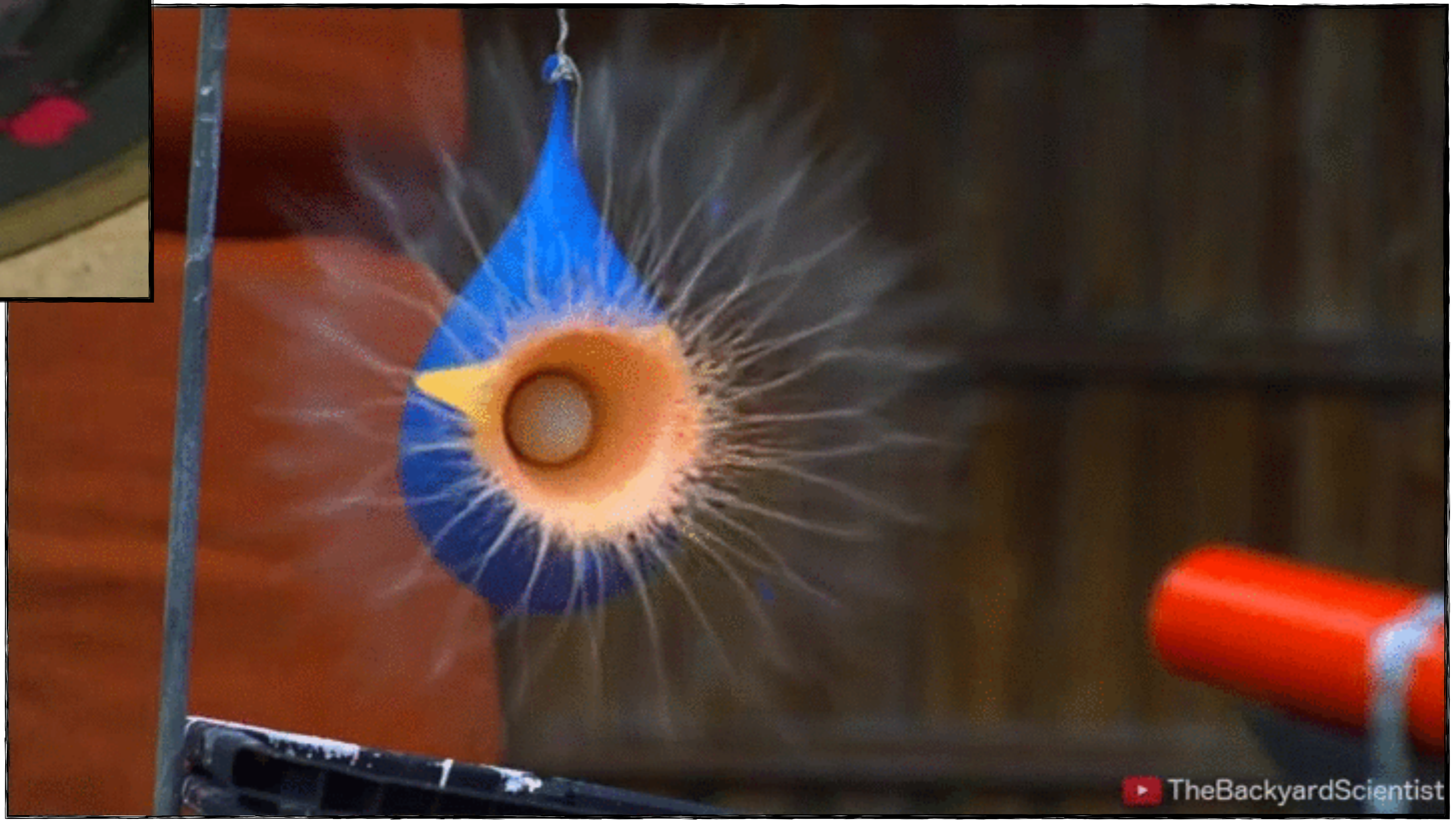


Violent increase in suspension viscosity!

Shear—Thickening Suspensions: Quite Fun!

credits: The Slow Mo Guys (Youtube).

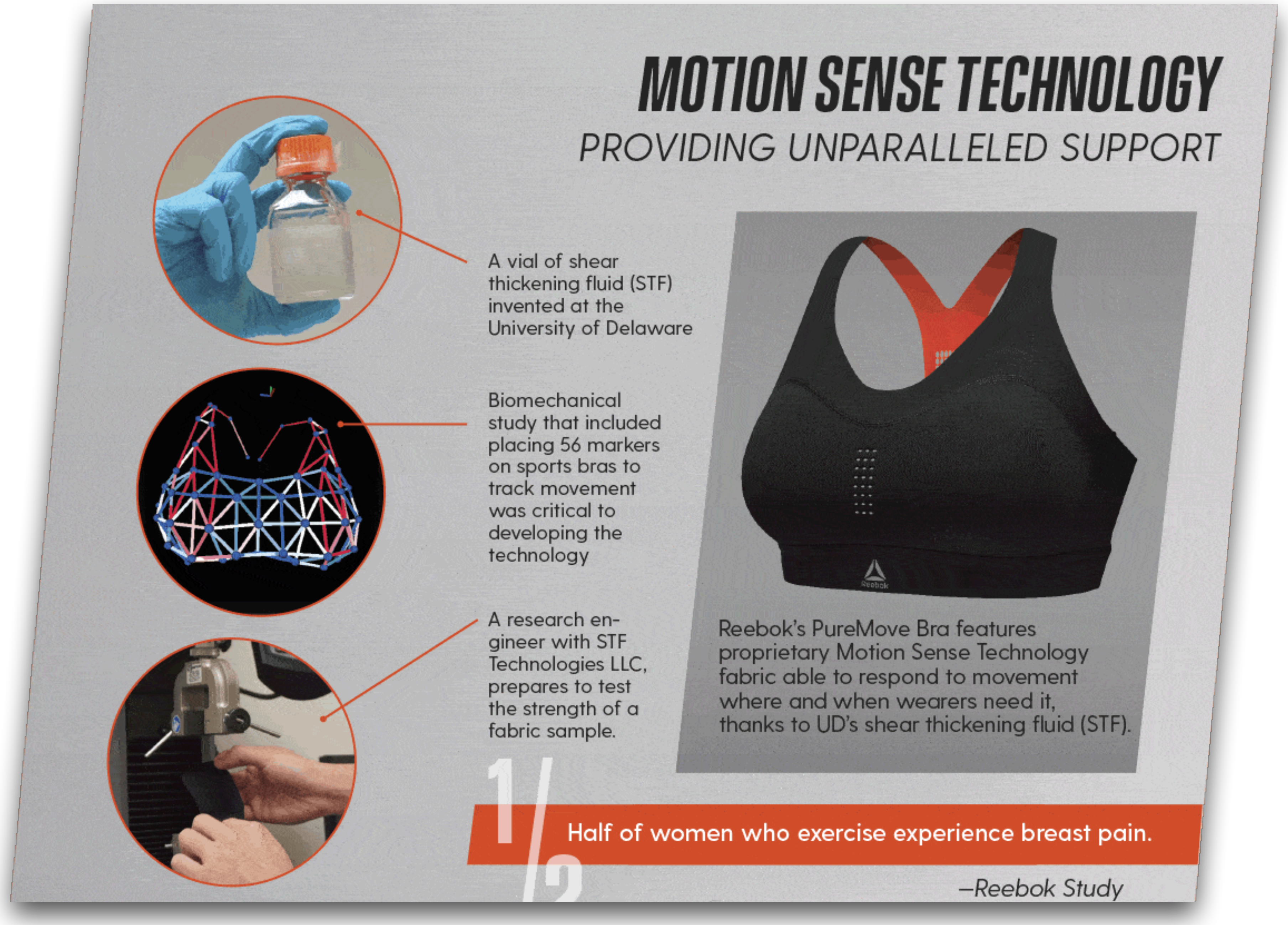
credits: The BackyardScientists (Youtube).



Some Applications...

Smart speedbumps

Sports bra



MOTION SENSE TECHNOLOGY

PROVIDING UNPARALLELED SUPPORT

A vial of shear thickening fluid (STF) invented at the University of Delaware

Biomechanical study that included placing 56 markers on sports bras to track movement was critical to developing the technology

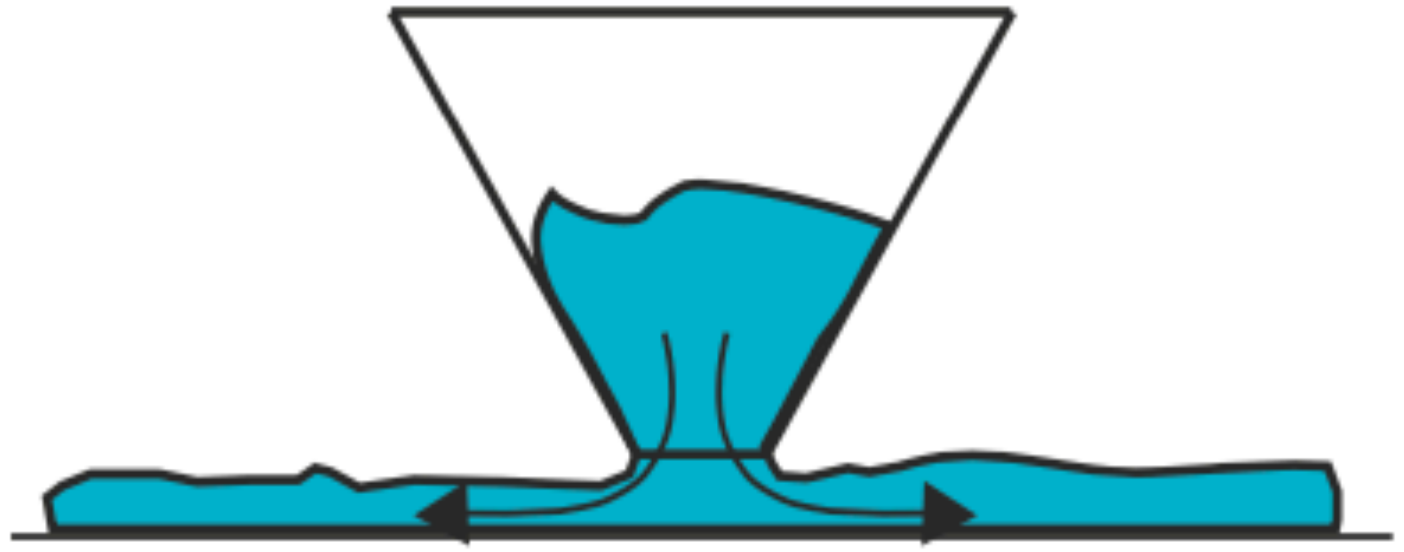
A research engineer with STF Technologies LLC, prepares to test the strength of a fabric sample.

Reebok's PureMove Bra features proprietary Motion Sense Technology fabric able to respond to movement where and when wearers need it, thanks to UD's shear thickening fluid (STF).

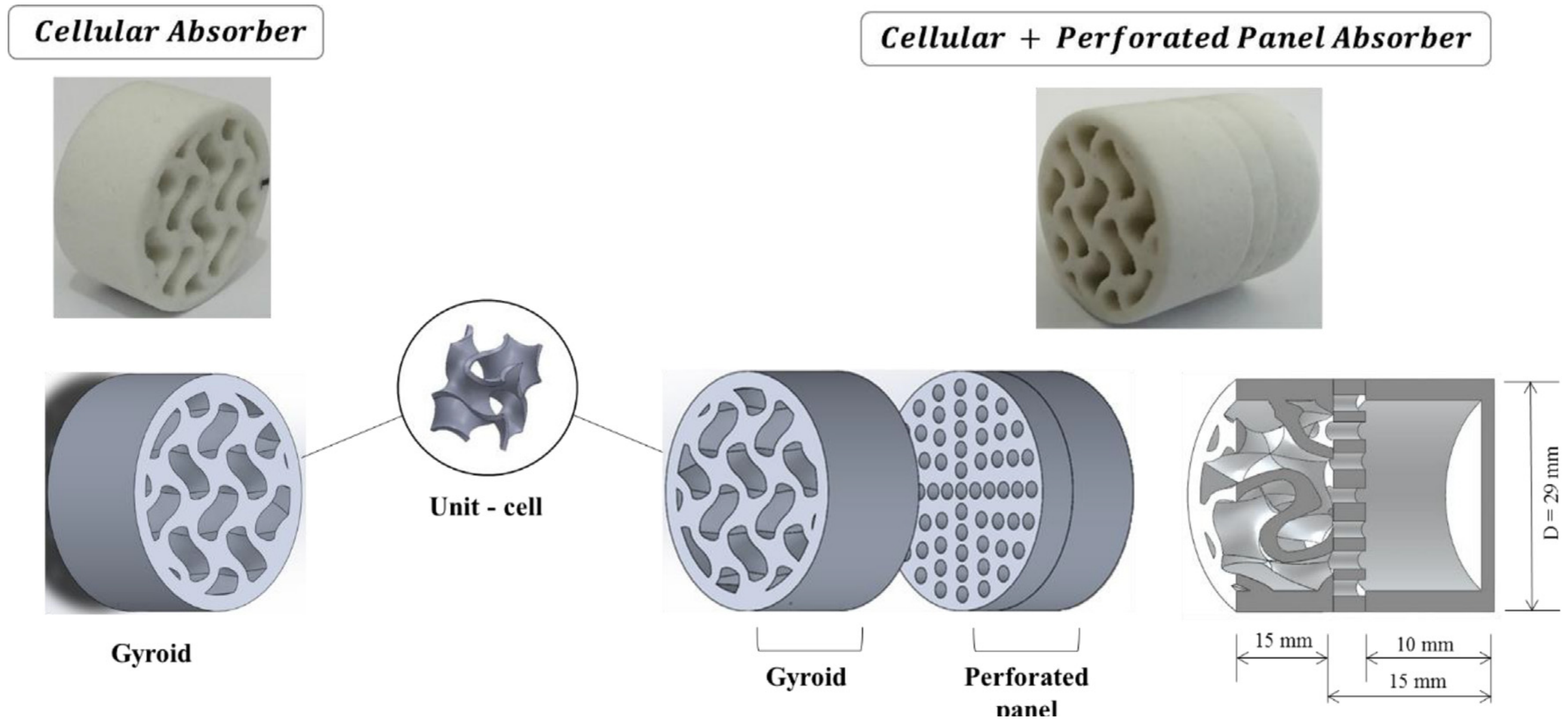
1/2 Half of women who exercise experience breast pain.

—Reebok Study

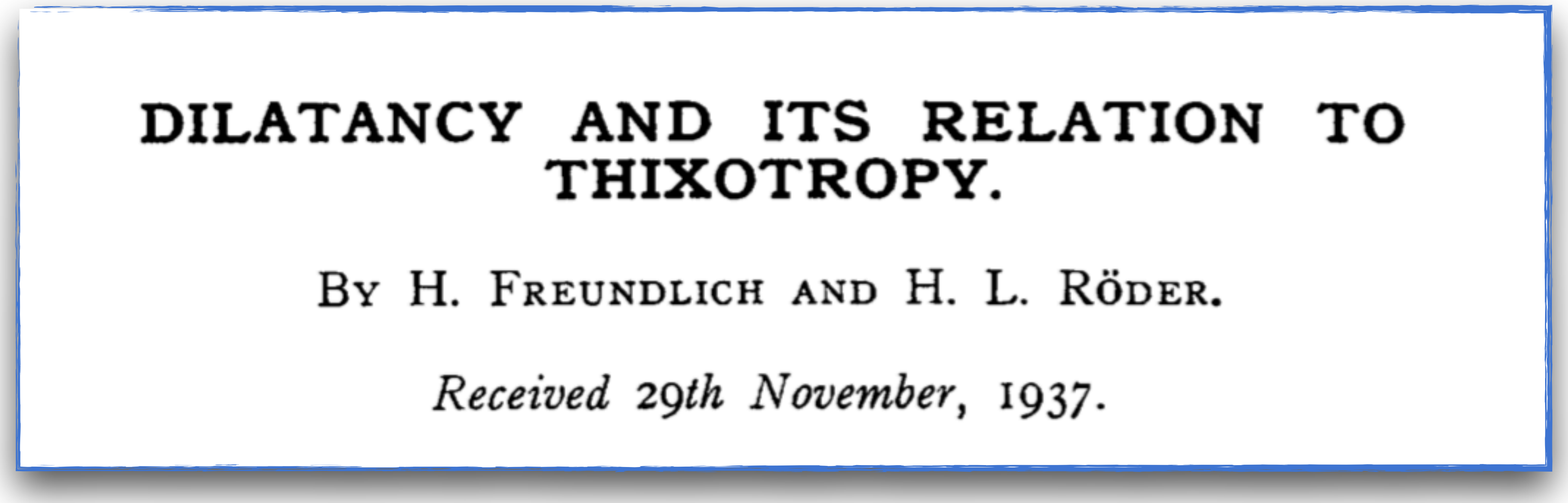
Modern concrete



Acoustic absorbers



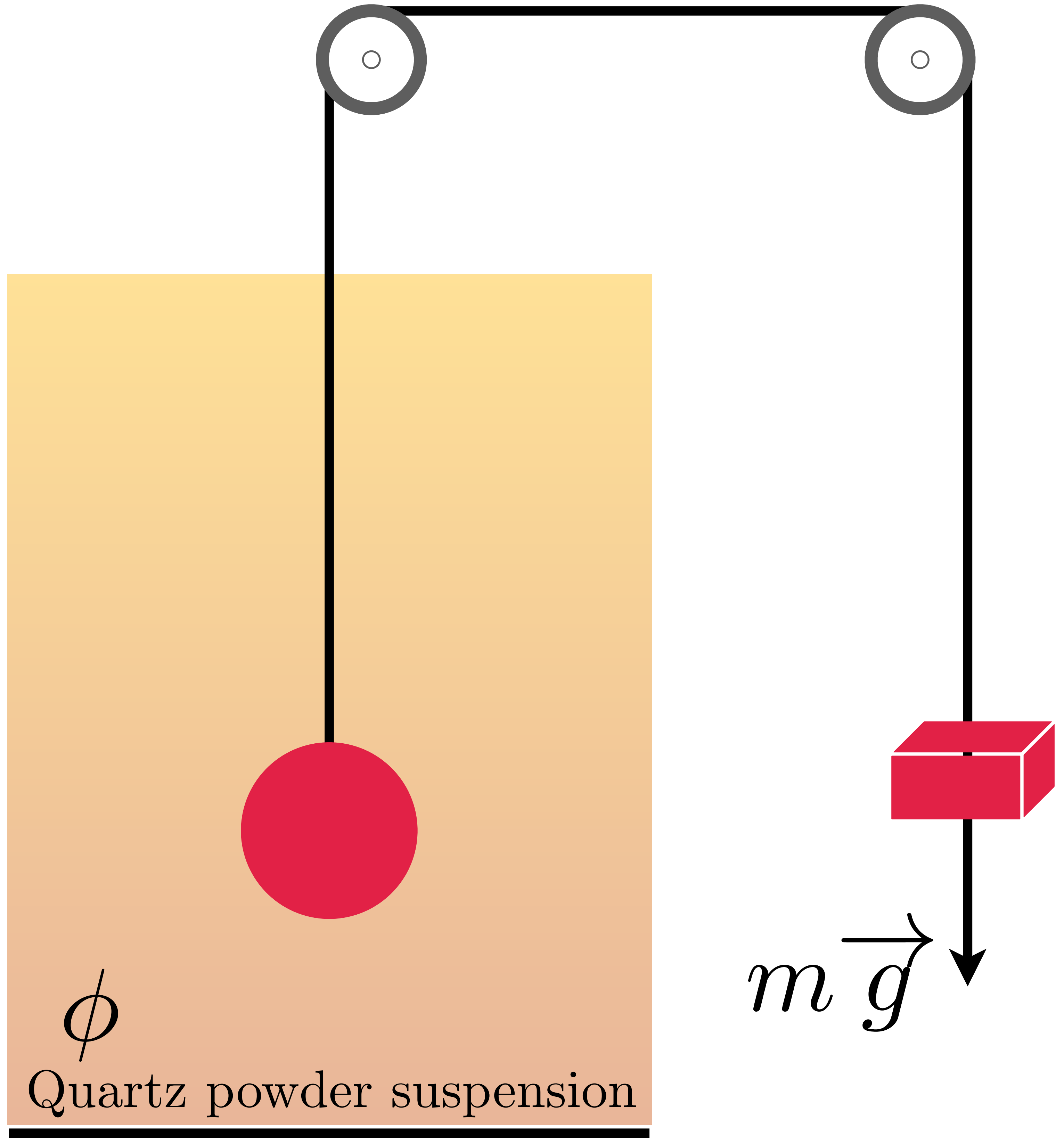
First Appearance in Scientific Journals



DILATANCY AND ITS RELATION TO THIXOTROPY.

By H. Freundlich and H. L. Röder.

Received 29th November, 1937.



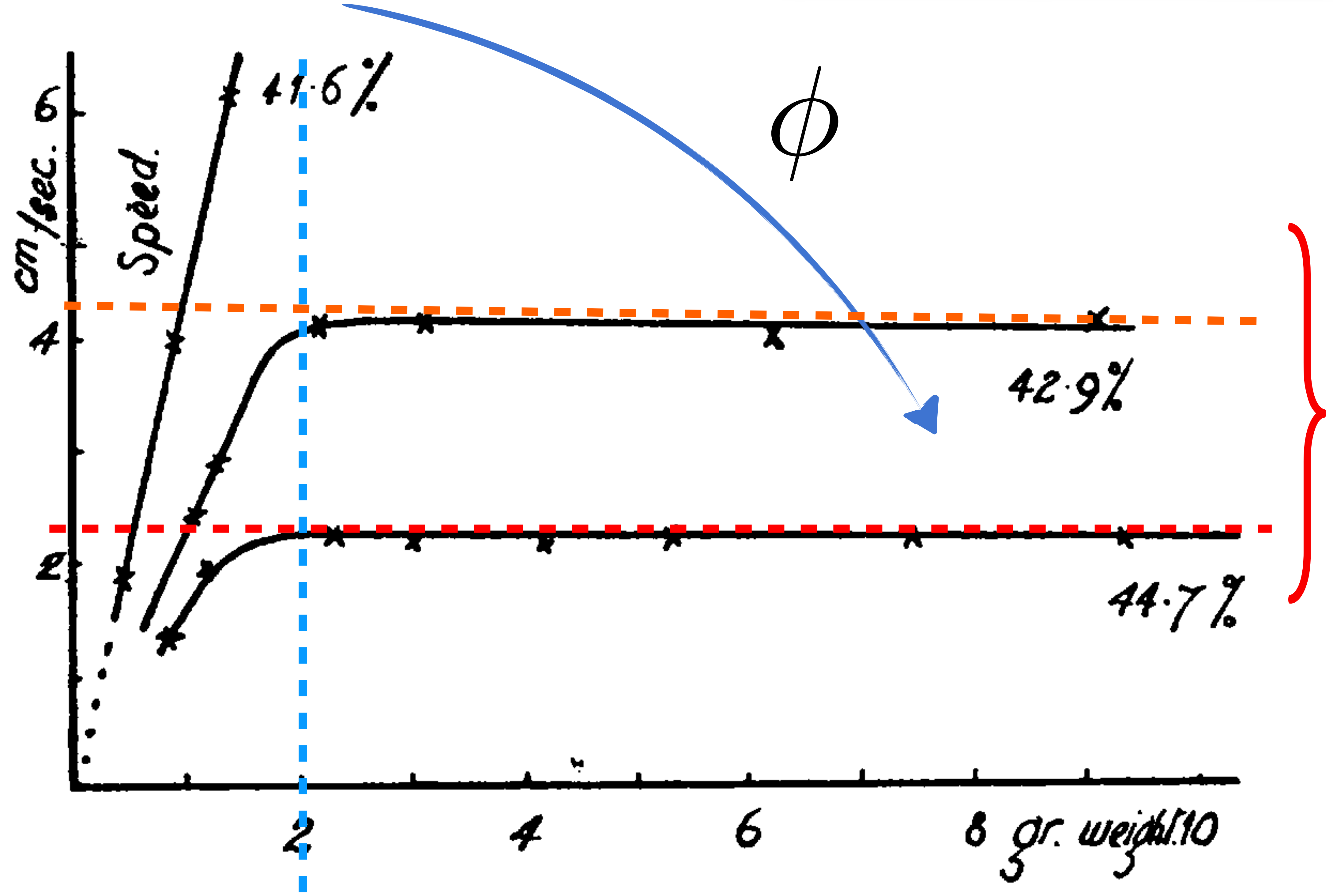


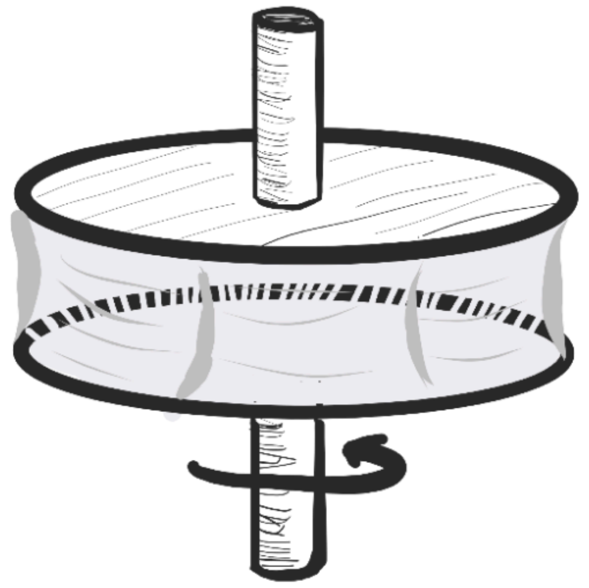
FIG. 2.

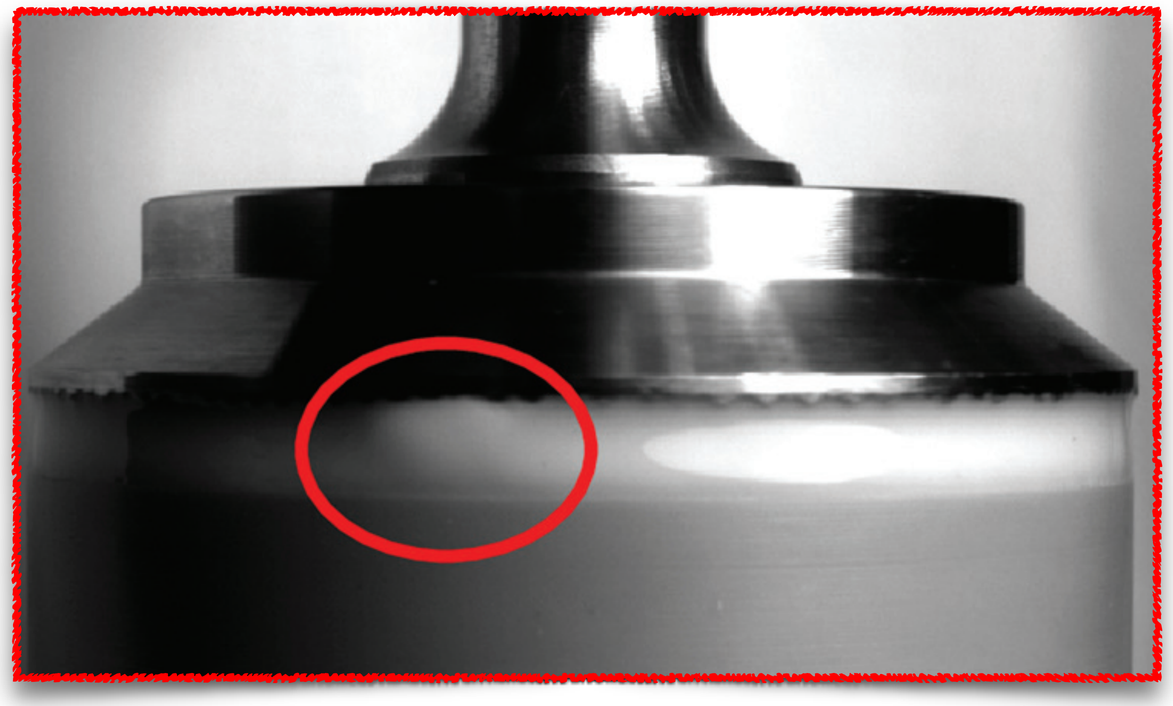
V_c

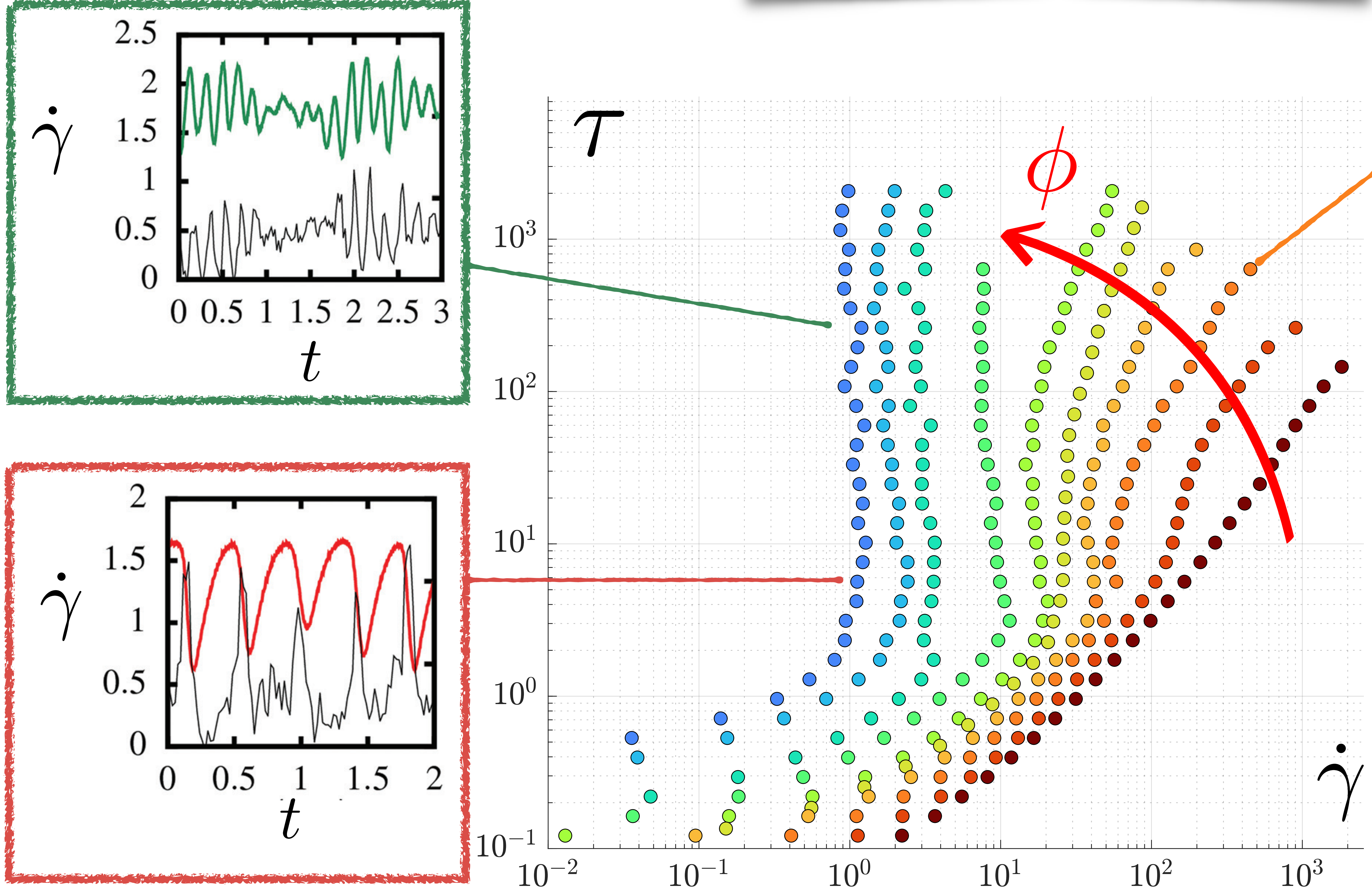
(Strongly dependent on ϕ)

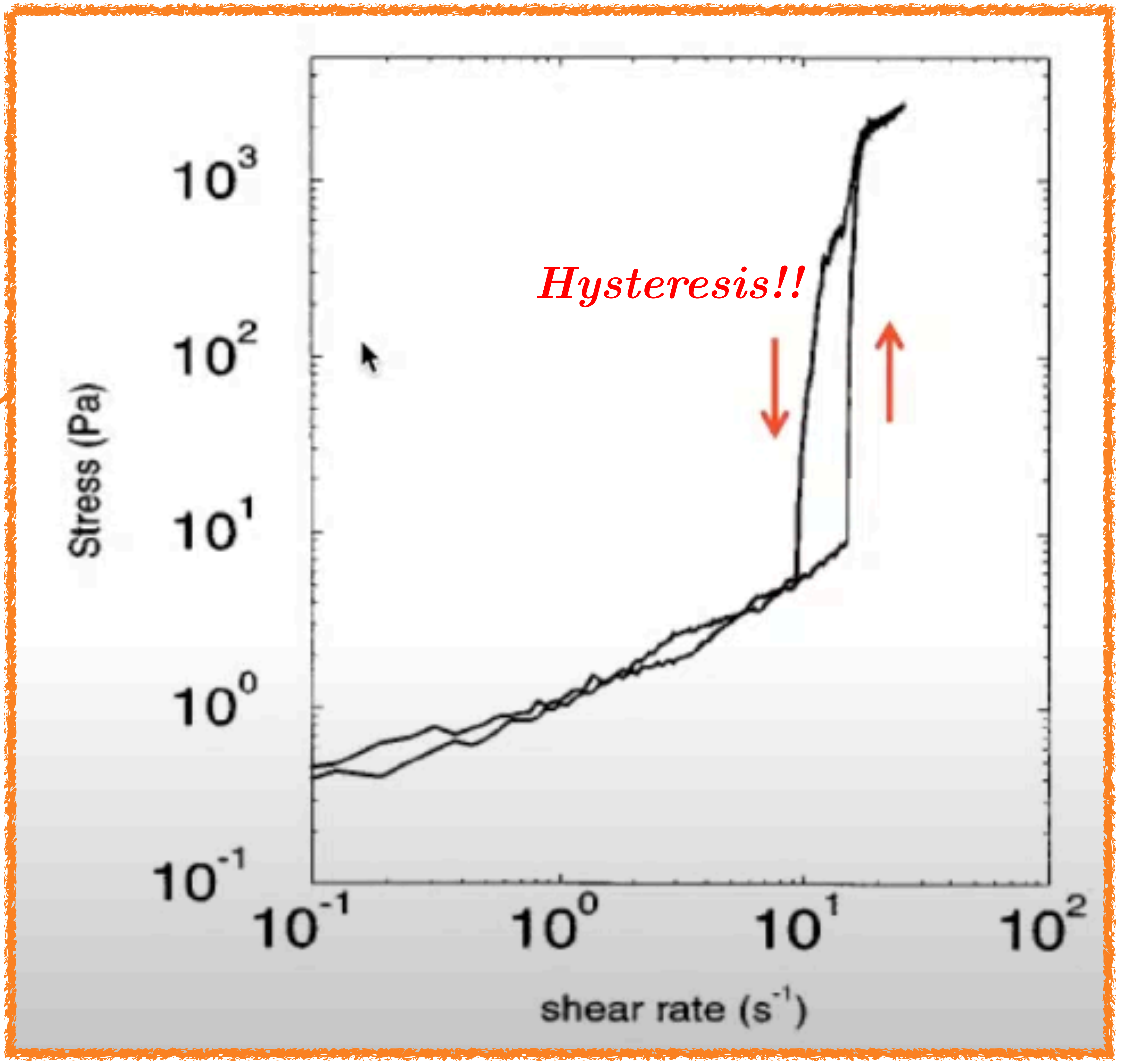
F_c (Very weak dependence on ϕ)

Experimental Observations: Volume—imposed Rheology









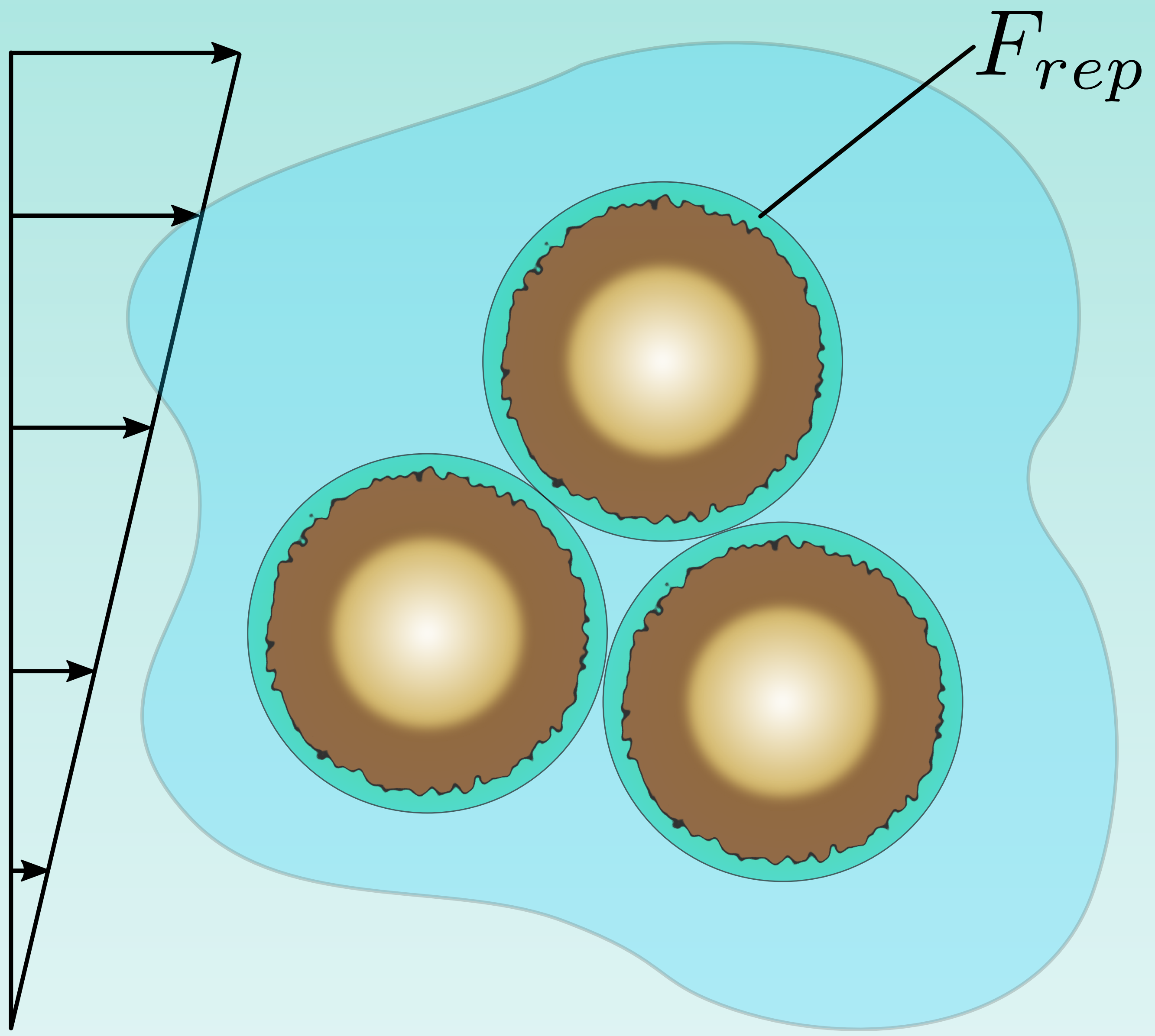
Hermes et al. JoR (2016)

Bender & Wagner JoR (1996)

Origin of Shear Thickening ?

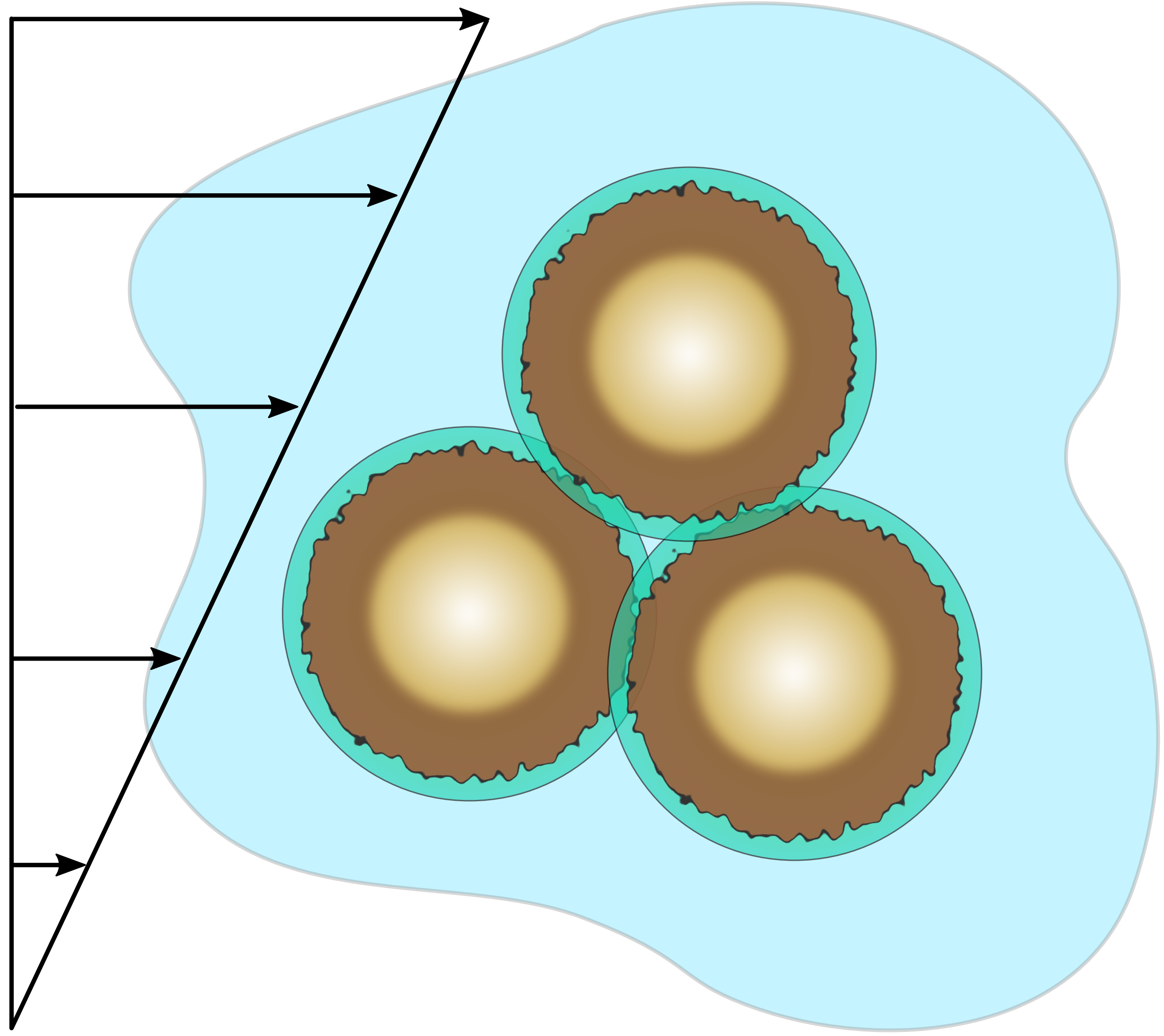
Key Idea: A short–range repulsive force between the grains (Seto *et al.* PRL 2013)

$$\boxed{P_{rep} \sim \frac{F_{rep}}{d^2}}$$



Low Stress $P < P_{rep}$

Frictionless



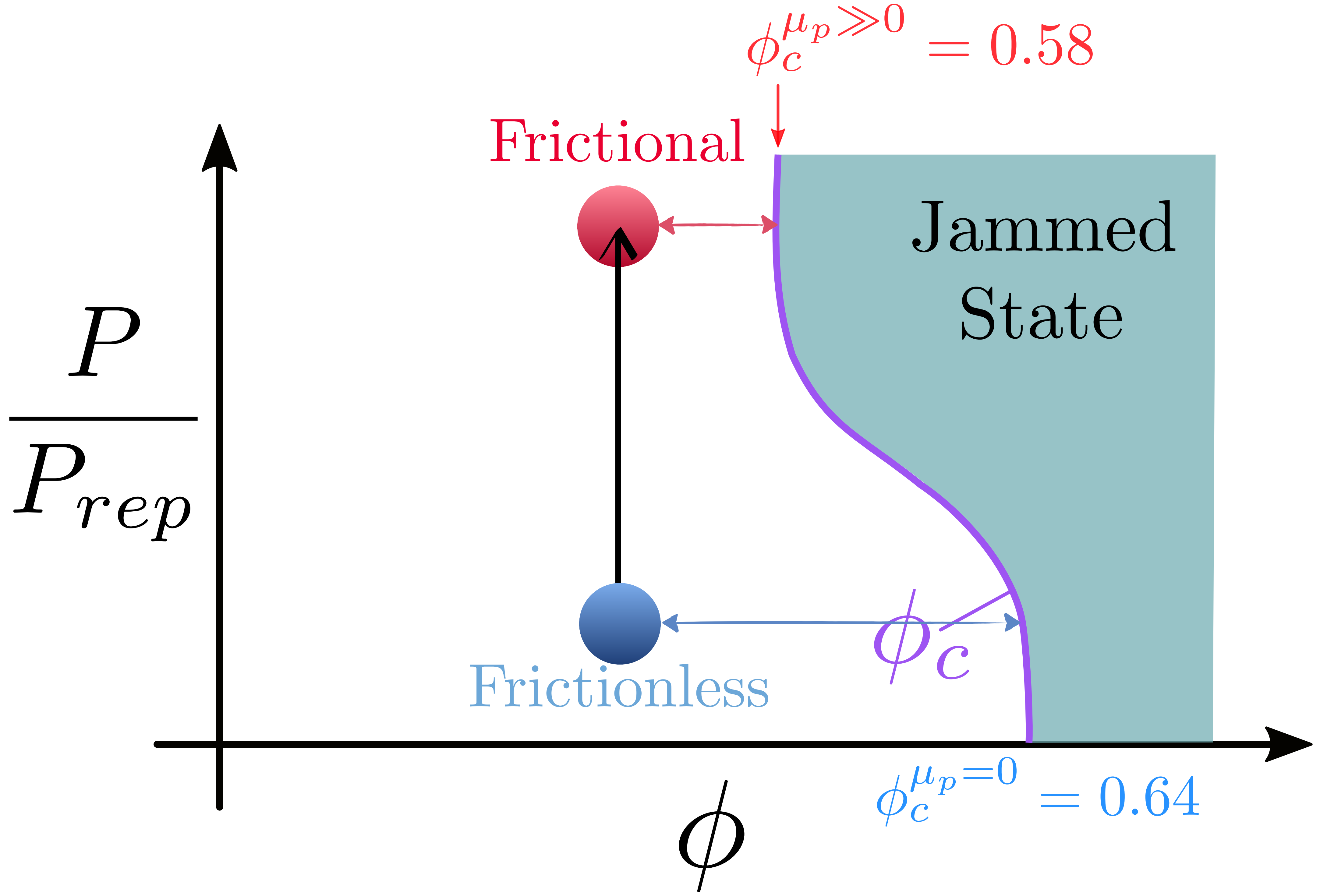
High Stress $P > P_{rep}$

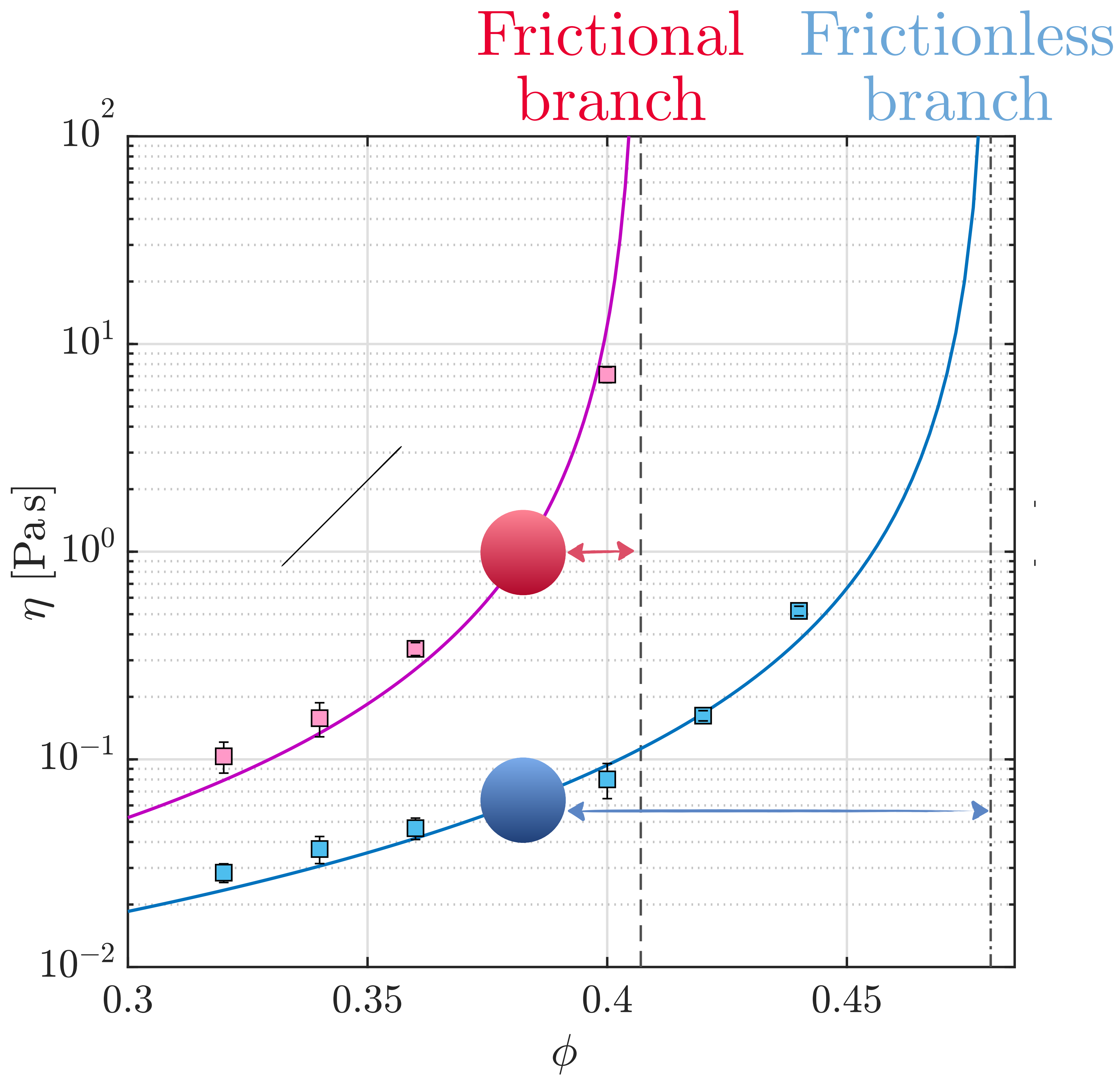
Frictional

→ “Activates” a FRICTIONLESS state at low stress!

Why is this so dramatic?

- The critical volume fraction φ_c is a function of the interparticle friction μ_p !





→ When the confining pressure *increases* the system gets *closer* to the jamming point!

→ Introduces a *STRESS* scale to the problem!

Frictional Transition Model:

(Wyart & Cates PRL 2014)

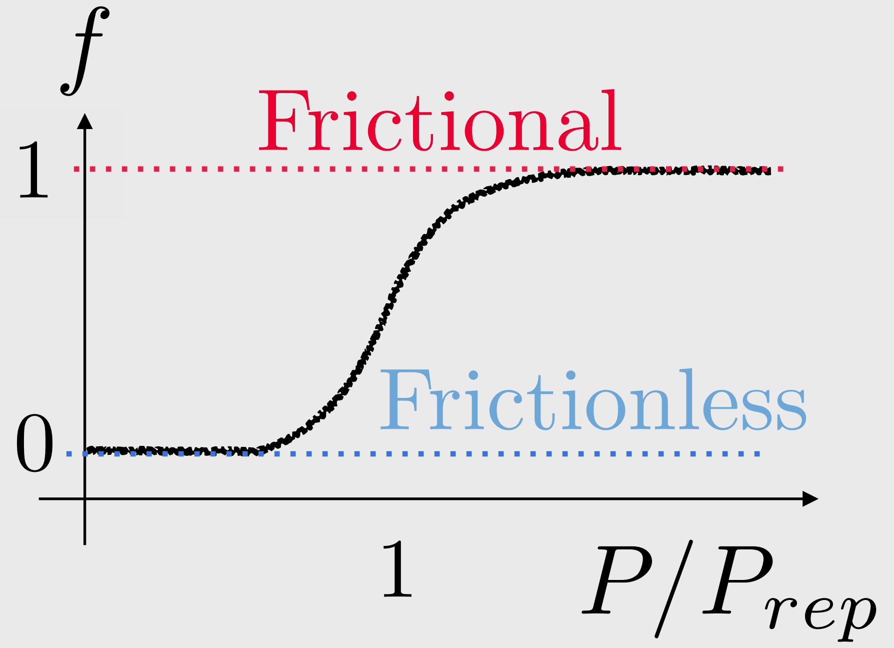
$$\frac{\eta_s}{\eta_f} = \left[\phi_c - \phi\right]^{-2} \quad \& \quad \phi_c\left(\frac{P}{P_{rep}}\right) = \phi_c^{\mu>0} f + (1-f)\phi_c^{\mu=0}$$

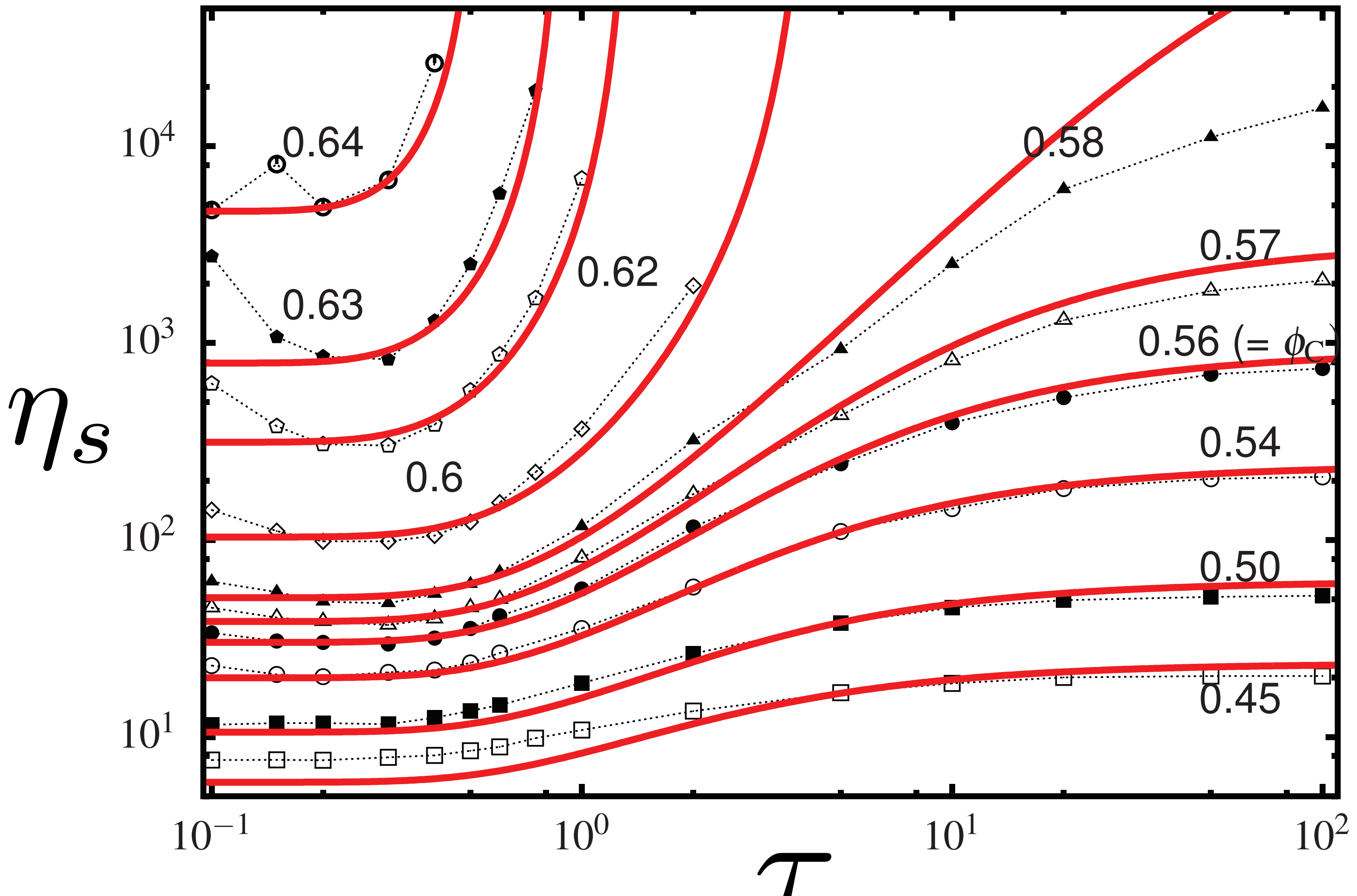
Order Parameter:

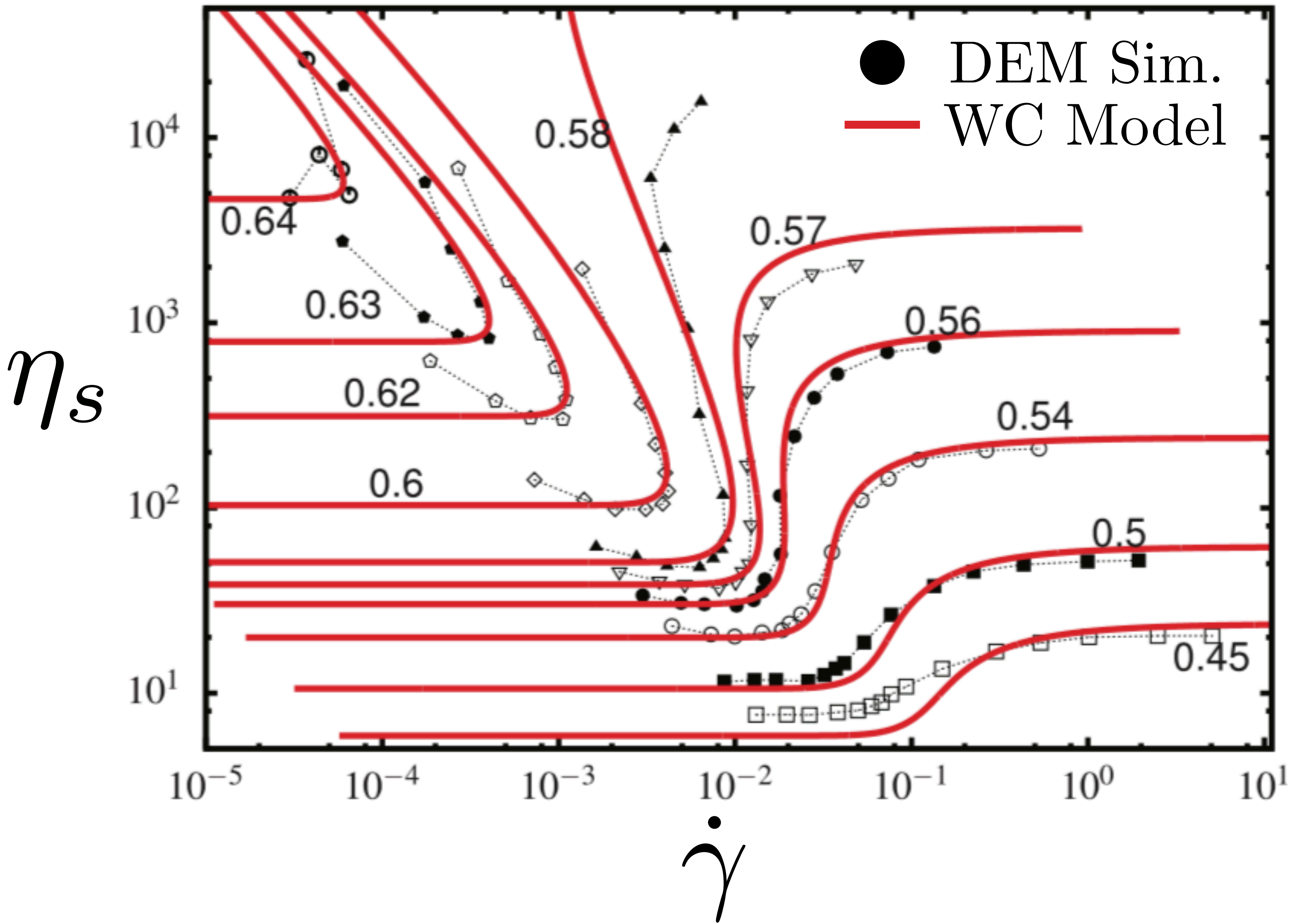
$f = 0 \rightarrow$ Frictionless when $P < P_{rep}$

$f = 1 \rightarrow$ Frictional when $P > P_{rep}$

Fraction of frictional contacts

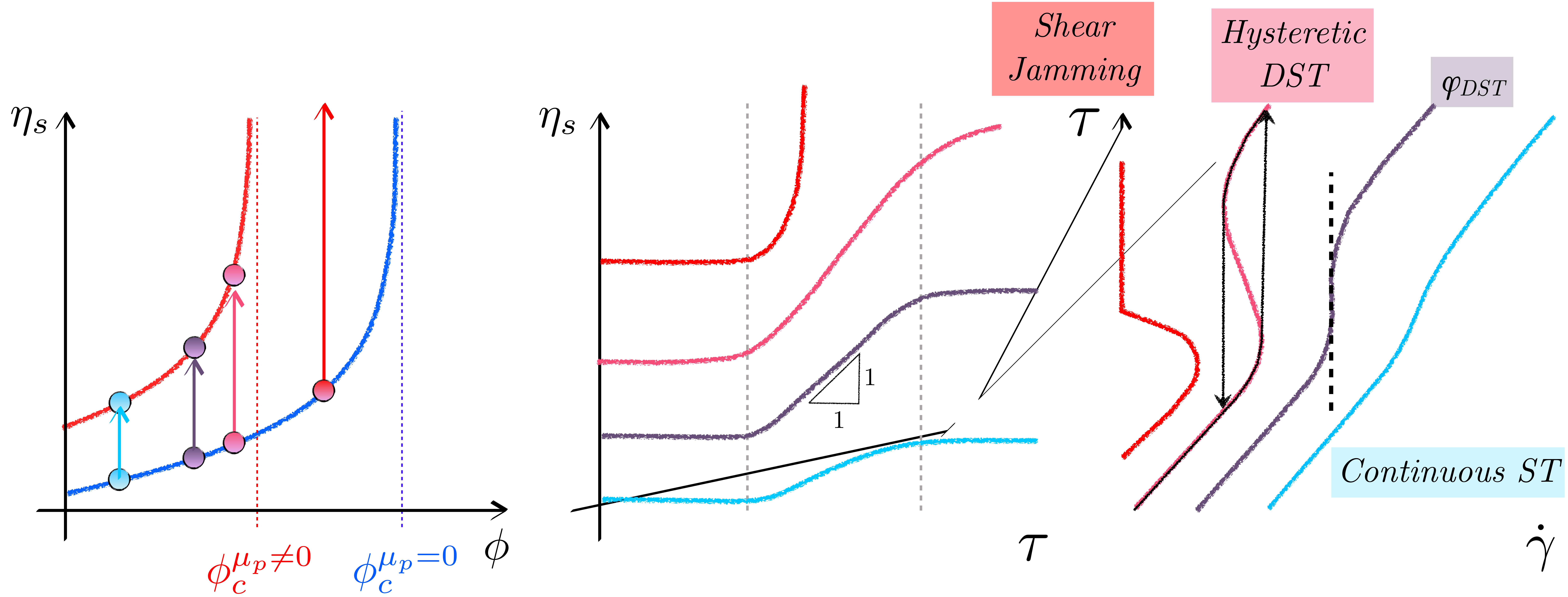






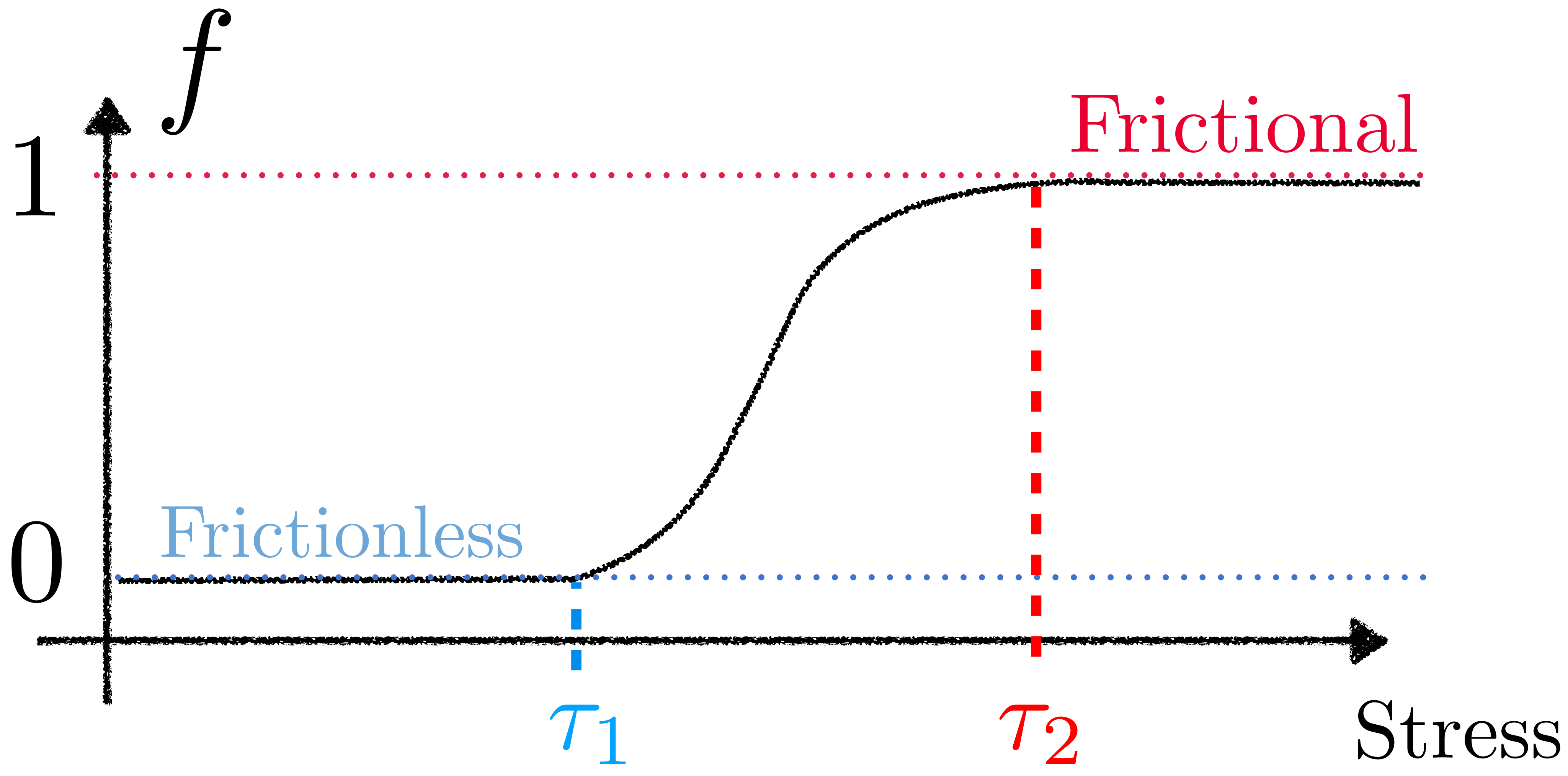
(Singh et al JoR 2018)

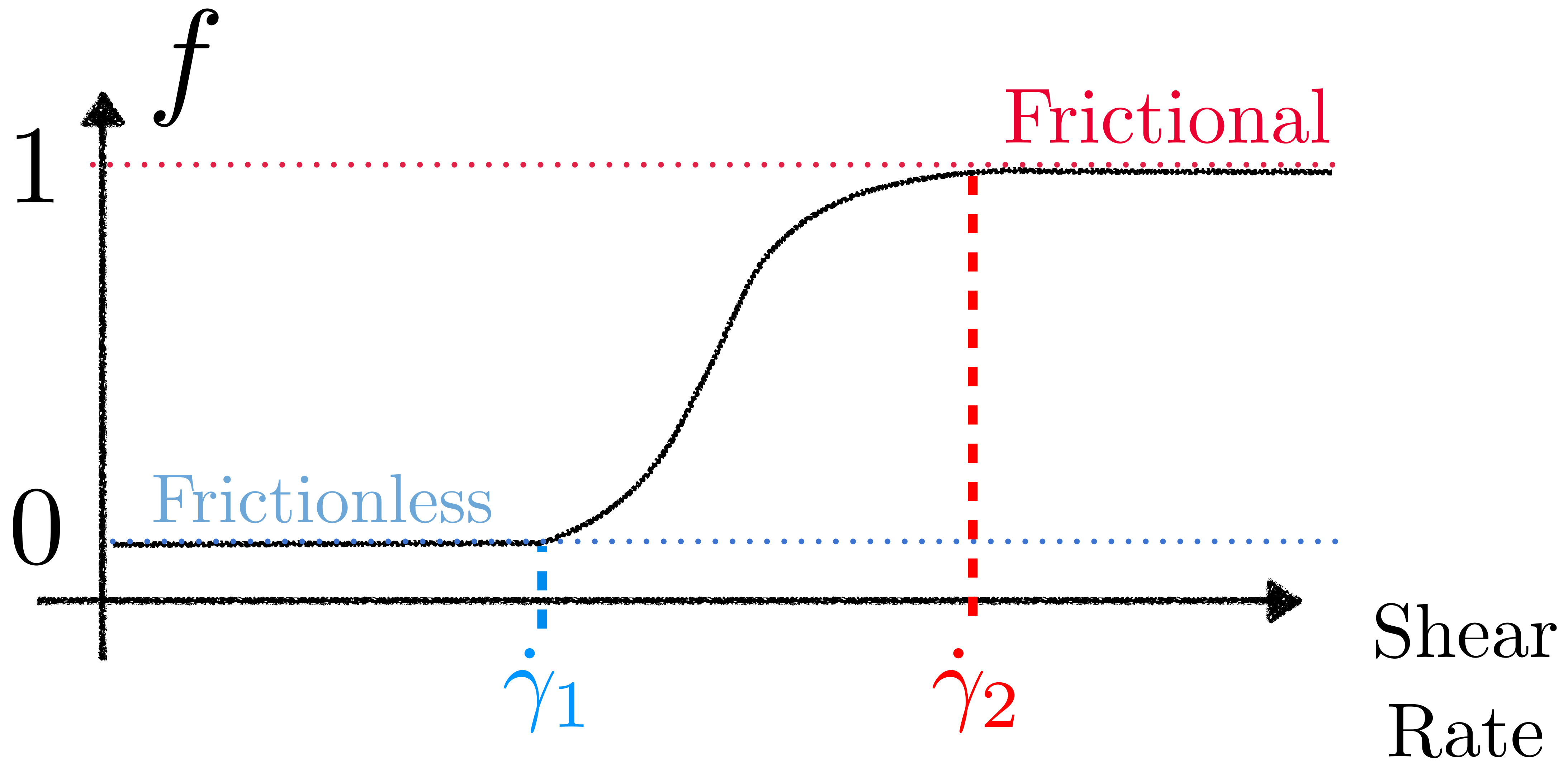
Frictional Transition Model



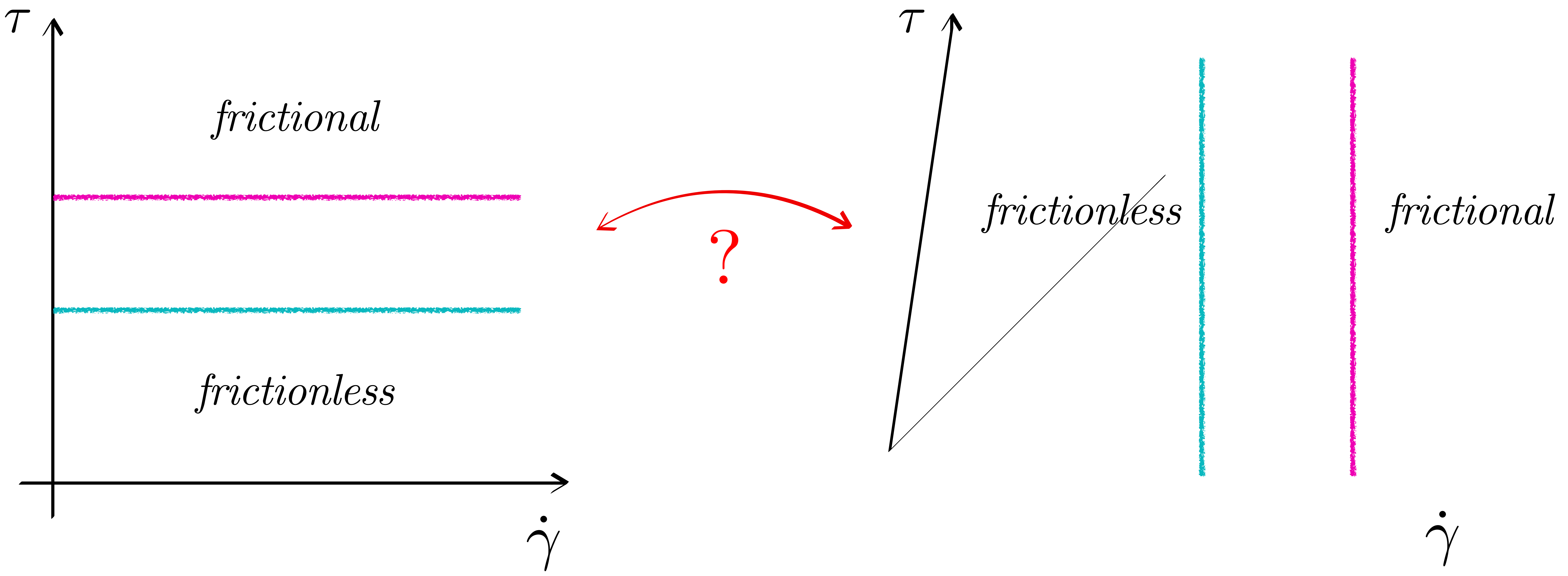
Frictional Transition Model

Does it give similar results?





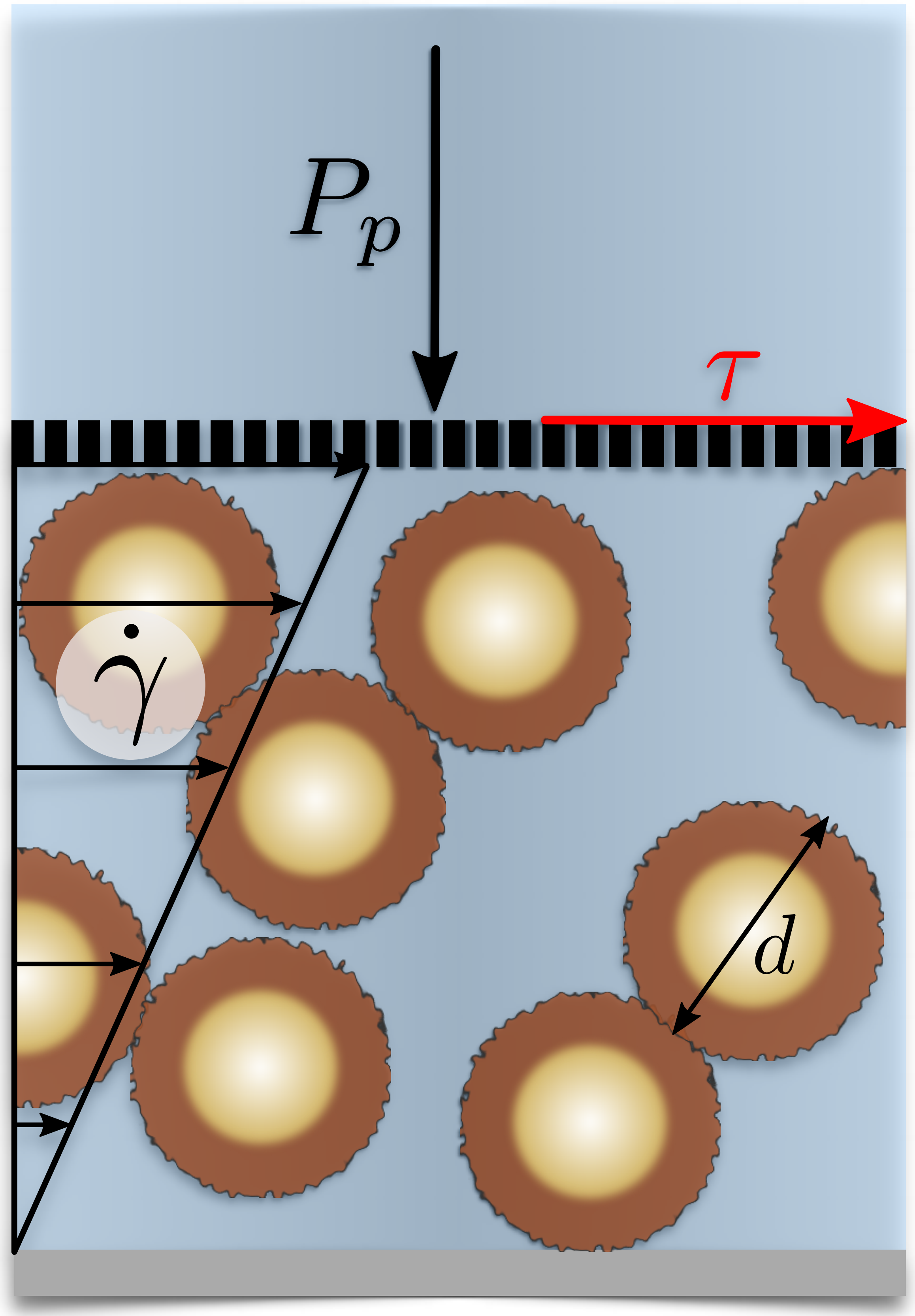
Frictional Transition Model



<u>Experimental Evidence of the Frictional Transition?</u>

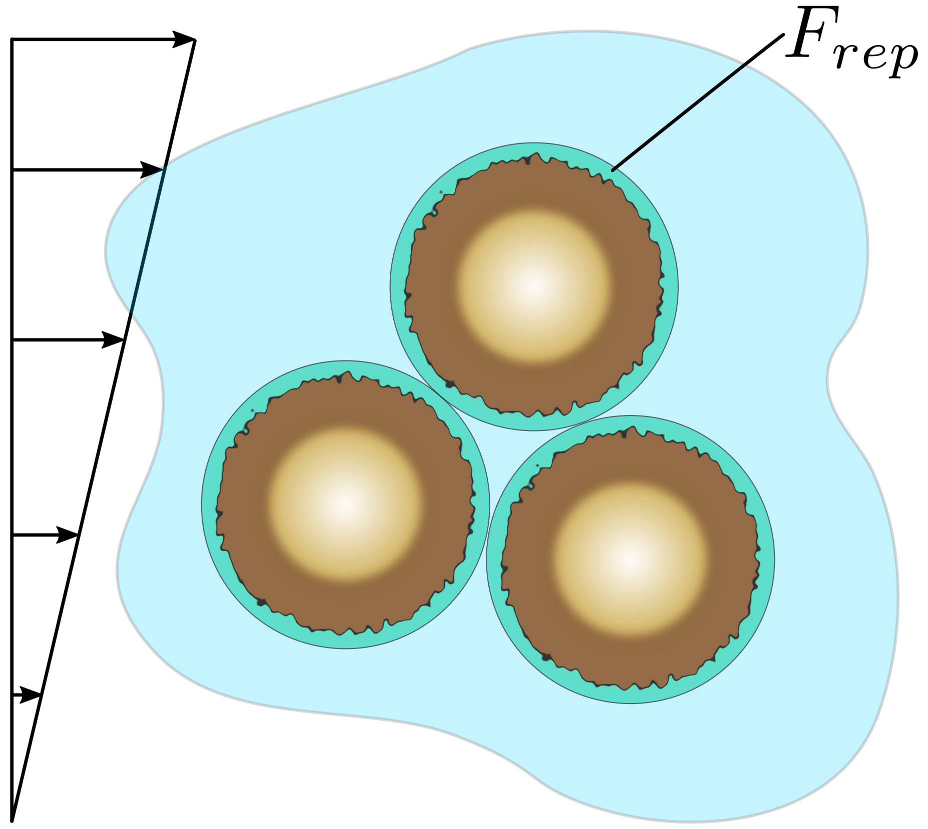
Experimental Evidence of the Frictional Transition?

Pressure-imposed Rheology



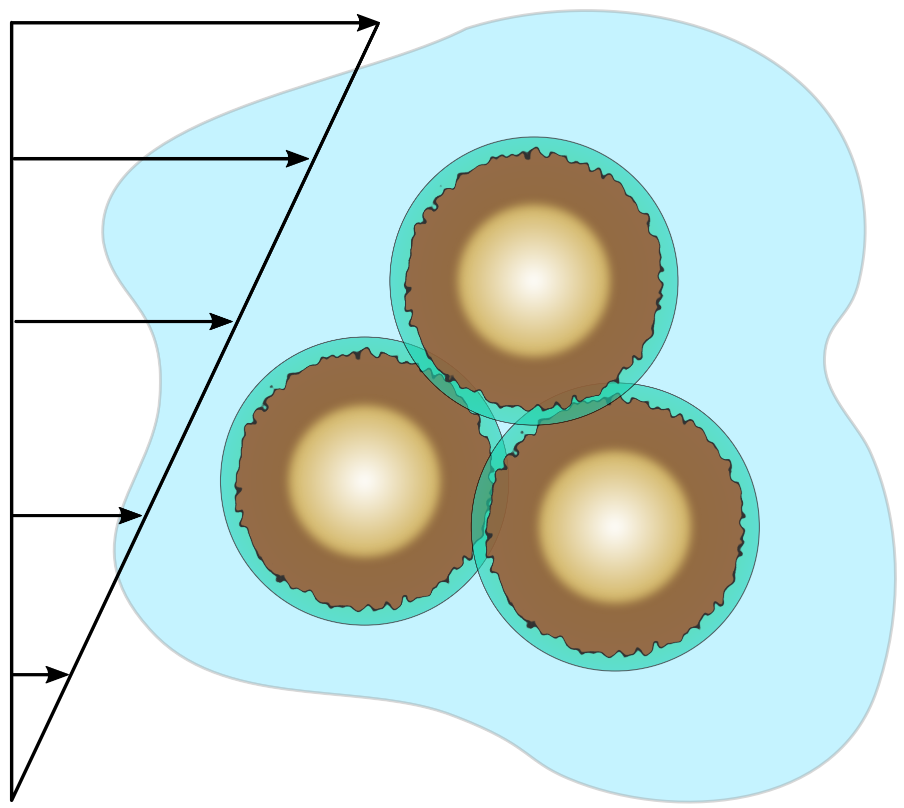
$$\mu = \frac{\tau}{P_p}$$

Frictional Transition:



Low Stress $P < P_{rep}$

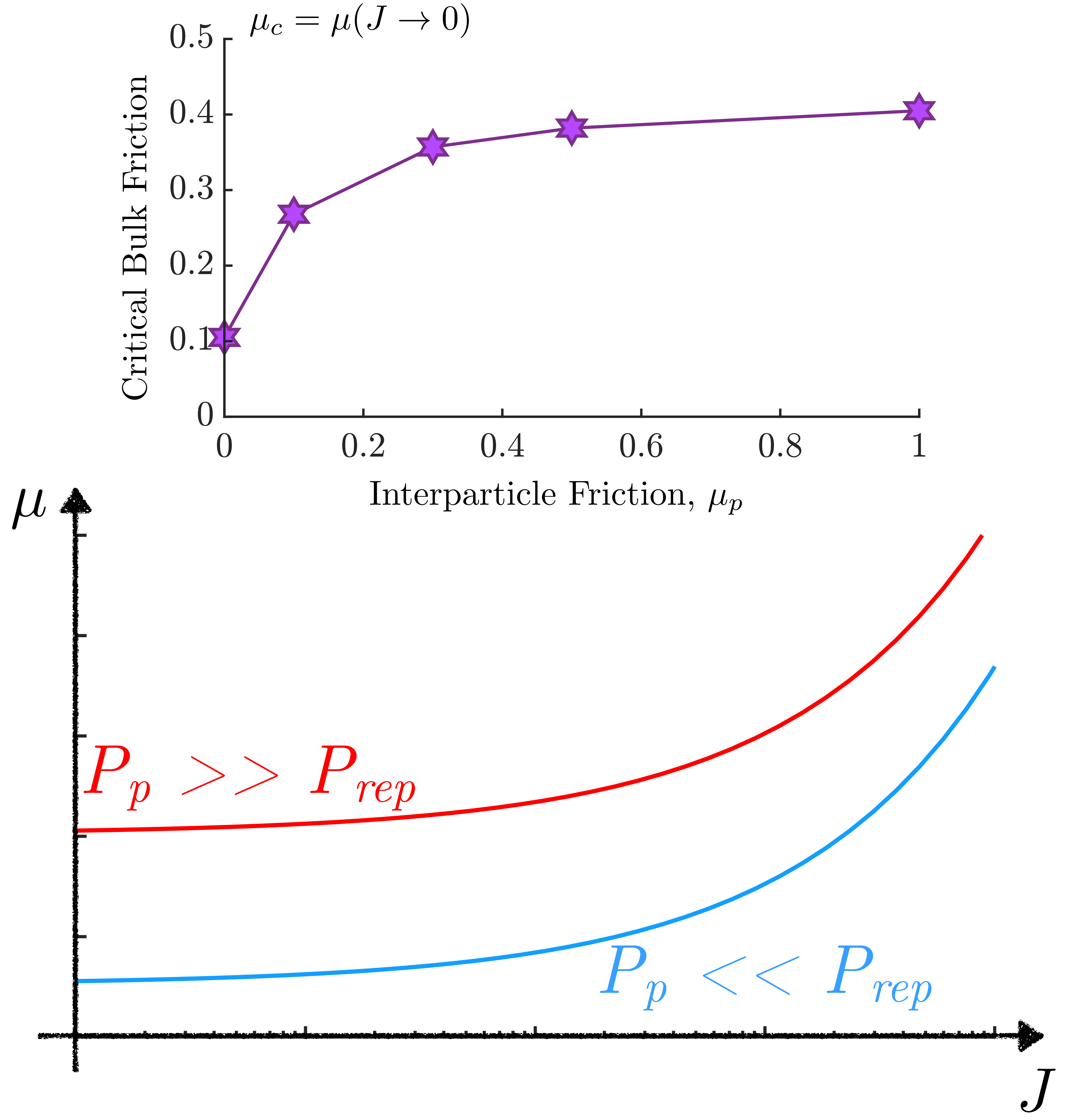
Frictionless



High Stress $P > P_{rep}$

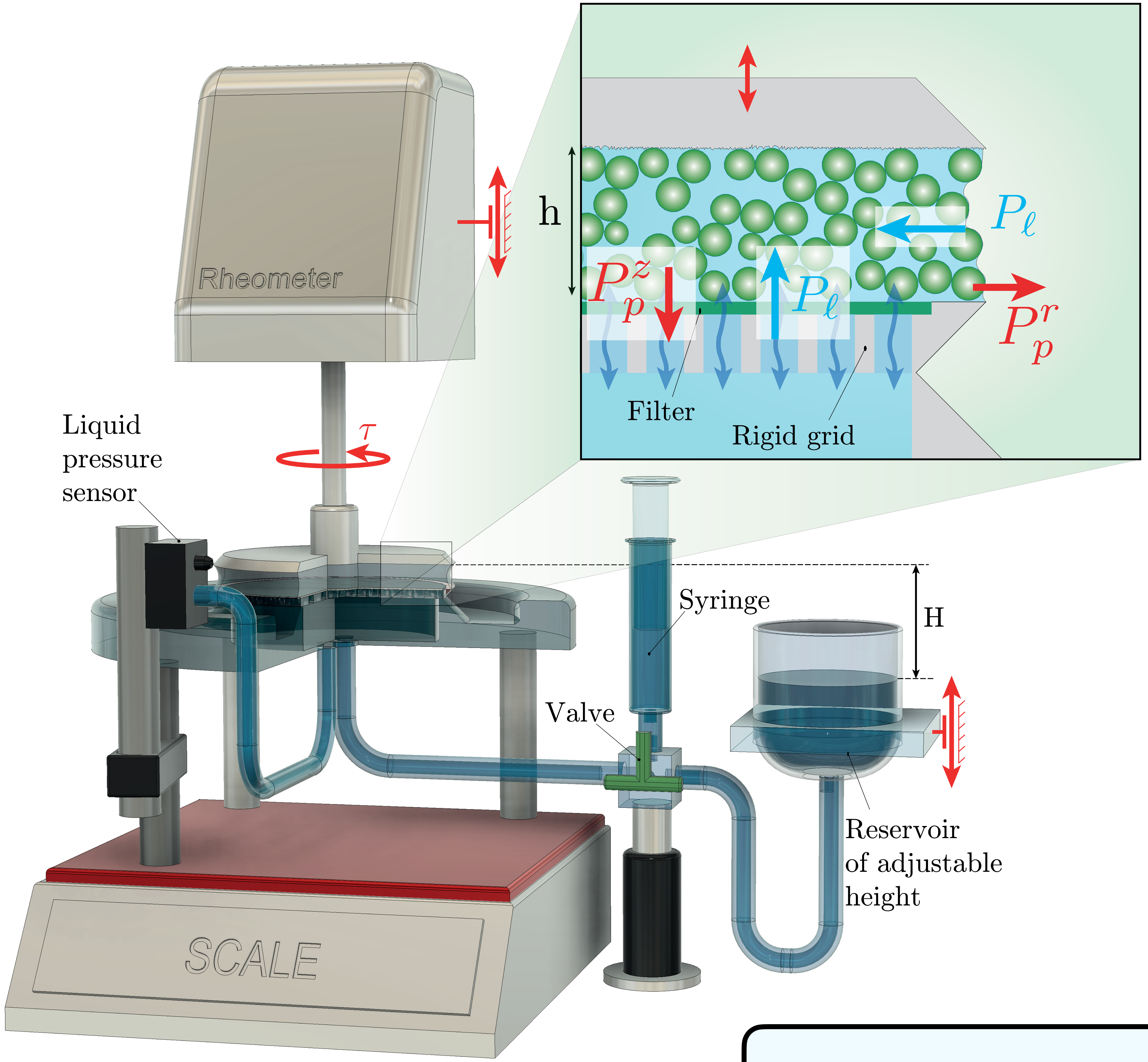
Frictional

What do we expect?



The Capillarytron

Etcheverry, Forterre & Metzger PRX (2022)



Imposed liquid pressure: $P_\ell = -\rho g H$

Radial force balance: $P_p^r = -P_\ell$

Capillary interface controls P_p

Vertical force balance: $mg = (P_p^z + P_\ell)\pi R^2$

Standard rheology:

$$\eta_s(\phi) = \frac{\tau}{\eta_f \dot{\gamma}} \quad \eta_n(\phi) = \frac{P_p^z}{\eta_f \dot{\gamma}}$$

Frictional rheology:

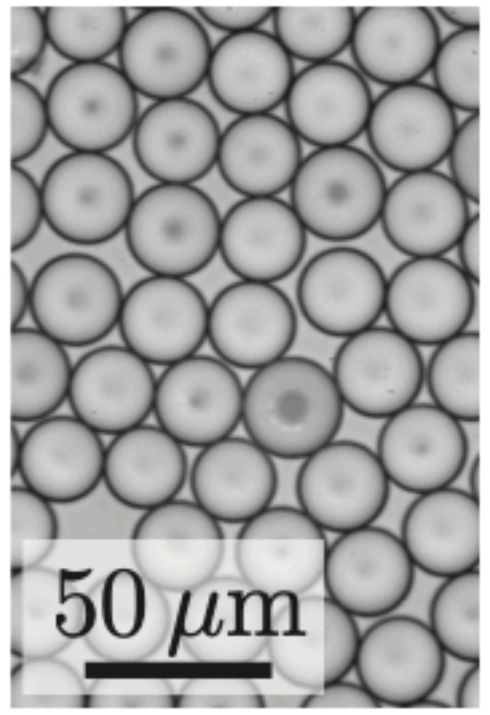
$$\boxed{\mu(J) = \frac{\tau}{P_p^z}} \quad \phi = \phi(J) \quad J = \frac{\eta_f \dot{\gamma}}{P_p^z}$$

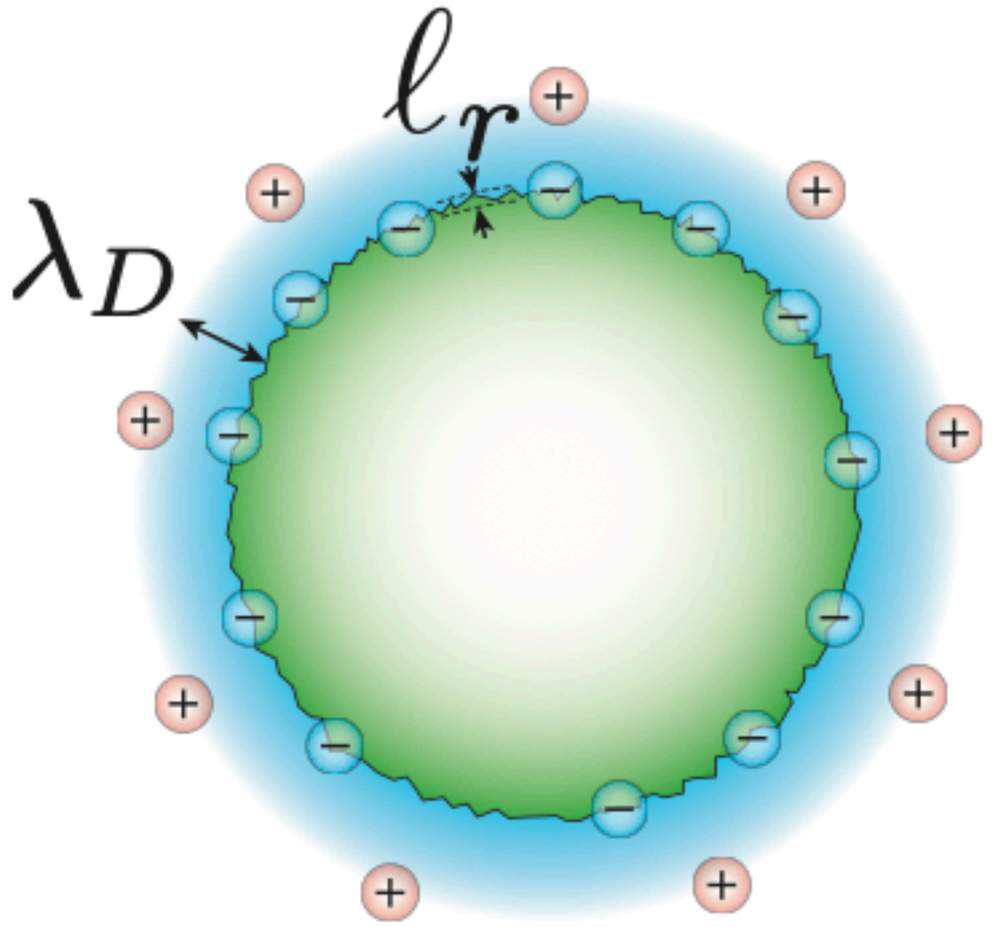
Able to perform both P_p—imposed or ϕ—imposed measurements !

A Model Shear-thickening Suspension

Clavaud et al. PNAS (2017)
Perrin et al. PRX (2019)
Perrin et al. PRL (2021)
Etcheverry, Forterre & Metzger PRX (2023)

Silica spheres
d=25 μm
Water+NaCl





Repulsive pressure:

$$P^\star = \frac{F_0 e^{-r/\lambda_D}}{d^2}$$

(~50 Pa in pure water)

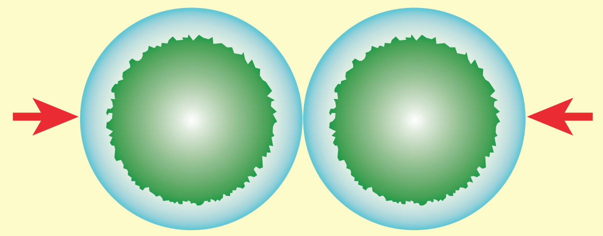
Debye length:

$$\lambda_d = \frac{0.304}{\sqrt{[NaCl]}} \text{ (nm)}$$

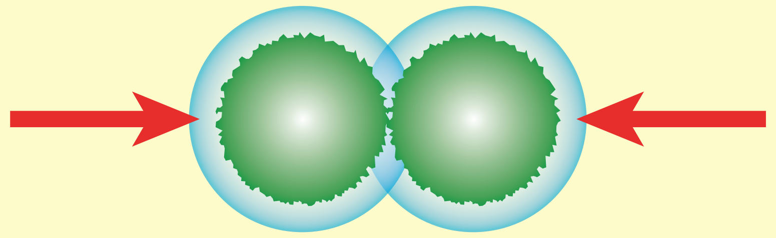
Frictional transition model:

(Seto et al. PRL 2013, Mari et al. JoR 2014, Wyart & Cates PRL 2014)

$$J = \frac{\eta\dot{\gamma}}{P_p} \qquad \frac{P_p}{P^\star}$$



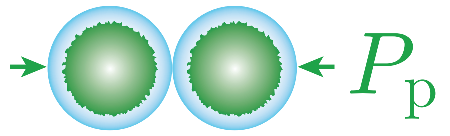
Frictionless

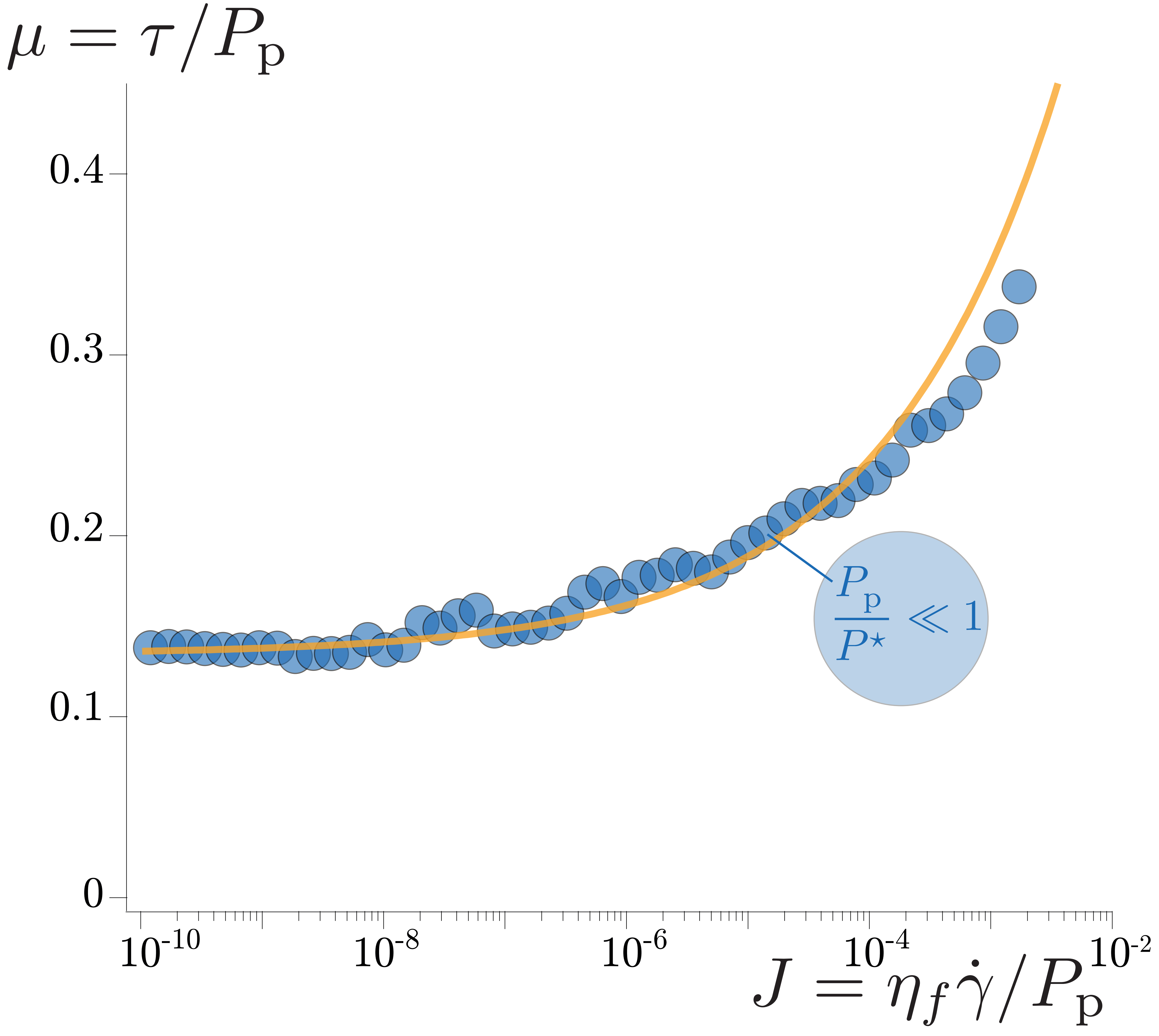


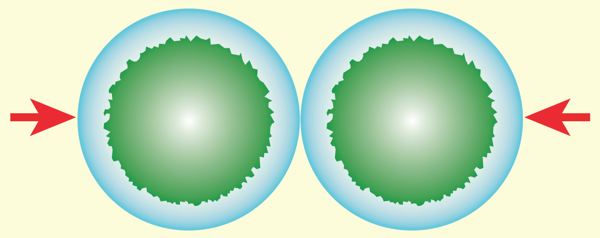
Frictional

Probing the Frictional Transition

Etcheverry, Forterre & Metzger PRX (2022)

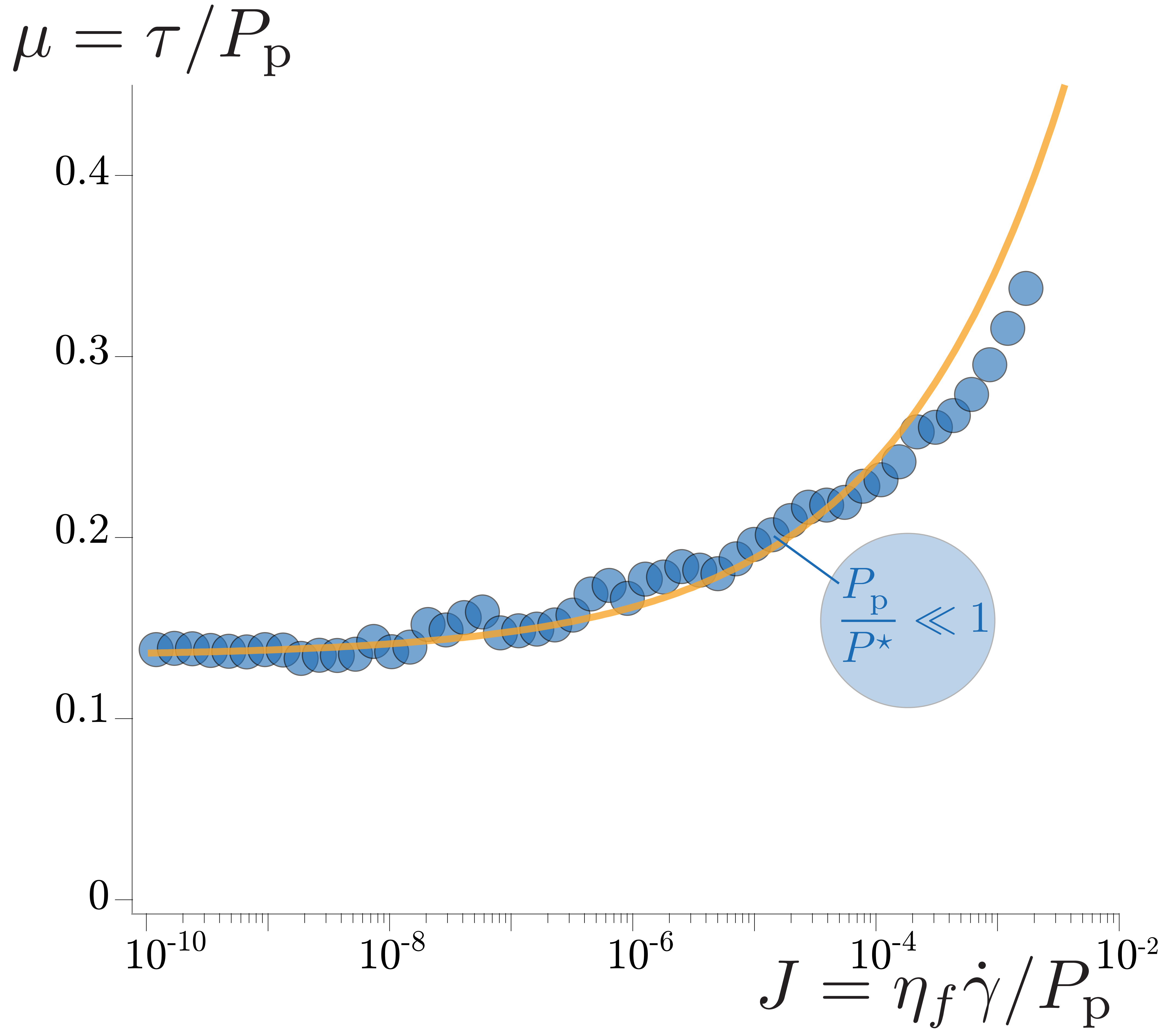
	$P_\mathrm{p}/P^\star$	Solvent	Frictional state
●	<<1	H_2O	Frictionless 



	Experiment	W&C Model
$\frac{P_p}{P^\star}$ <<1	$\mu = 0.13 + 1.7J^{0.3}$ Frictionless particles!	 Frictionless

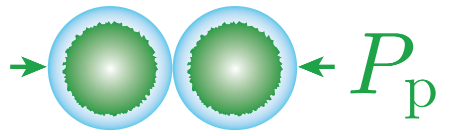
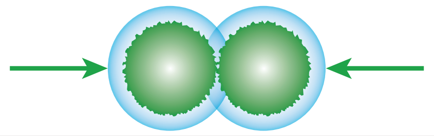
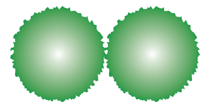
Probing the Frictional Transition

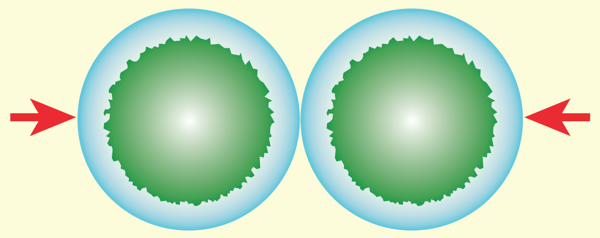
Etcheverry, Forterre & Metzger PRX (2022)



↗ Pp

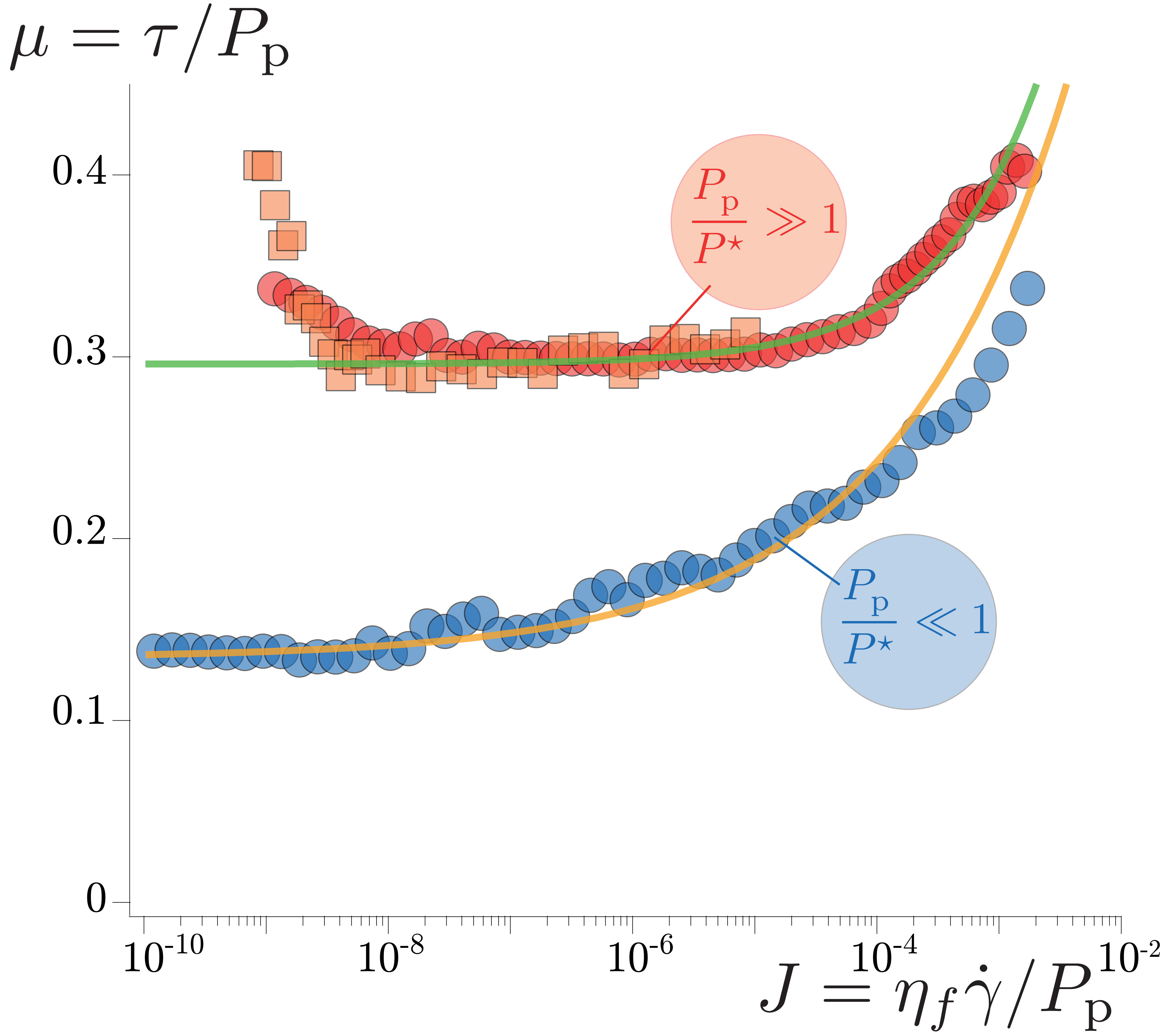
↘ P^*

	$P_\mathrm{p}/P^\star$	Solvent	Frictional state
●	<<1	H_2O	Frictionless 
■	>>1	H_2O	Frictional 
●	>>1	H_2O+NaCl	Frictional 

	Experiment	W&C Model
$\frac{P_p}{P^\star}$ <<1	$\mu = 0.13 + 1.7J^{0.3}$ → Frictionless particles!	 Frictionless

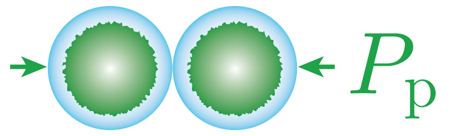
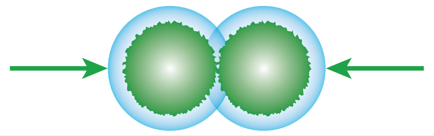
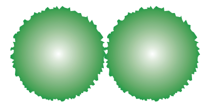
Probing the Frictional Transition

Etcheverry, Forterre & Metzger PRX (2022)



↗ Pp

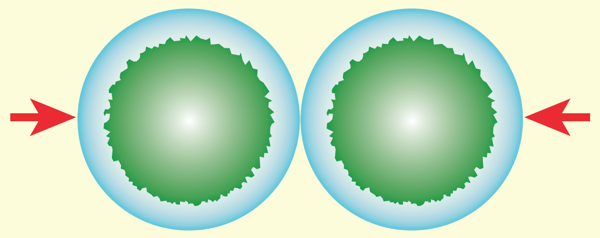
↘ P^*

	$P_\mathrm{p}/P^\star$	Solvent	Frictional state
●	<<1	H_2O	**Frictionless** 
■	>>1	H_2O	**Frictional** 
●	>>1	H_2O+NaCl	**Frictional** 

Experiment | W&C Model

$\frac{P_p}{P^\star}$ <<1 $\mu = 0.13 + 1.7J^{0.3}$ →

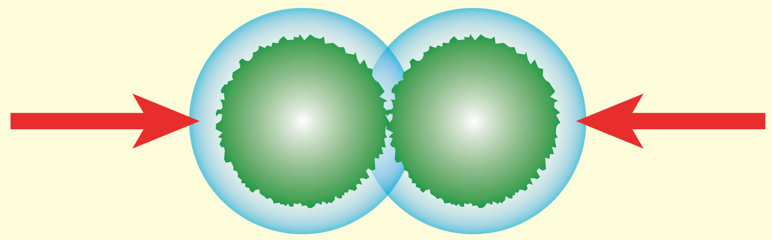
Frictionless particles!



Frictionless

$\frac{P_p}{P^\star}$ >>1 $\mu = 0.3 + 3.2J^{0.5}$ →

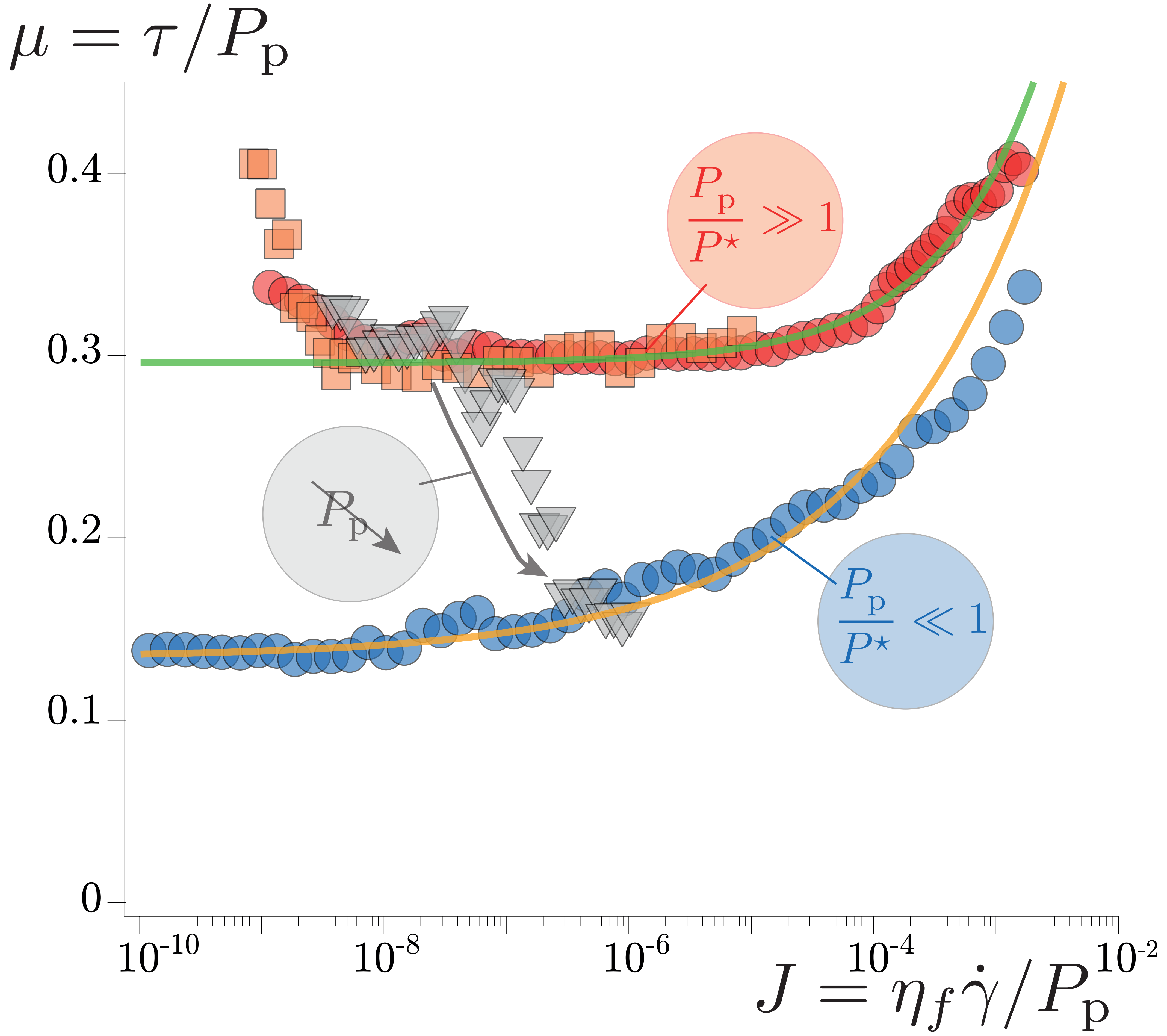
Frictional particles!

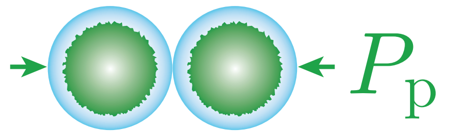
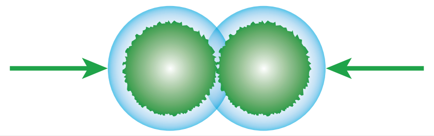
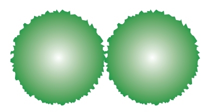
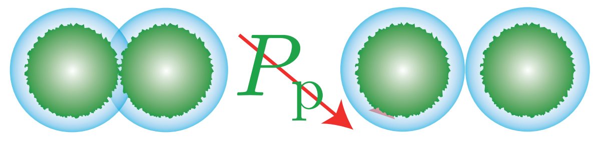


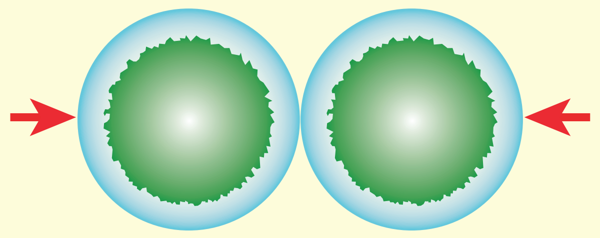
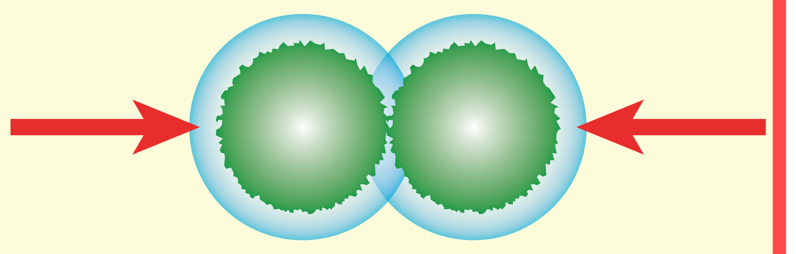
Frictional

Probing the Frictional Transition

Etcheverry, Forterre & Metzger PRX (2022)

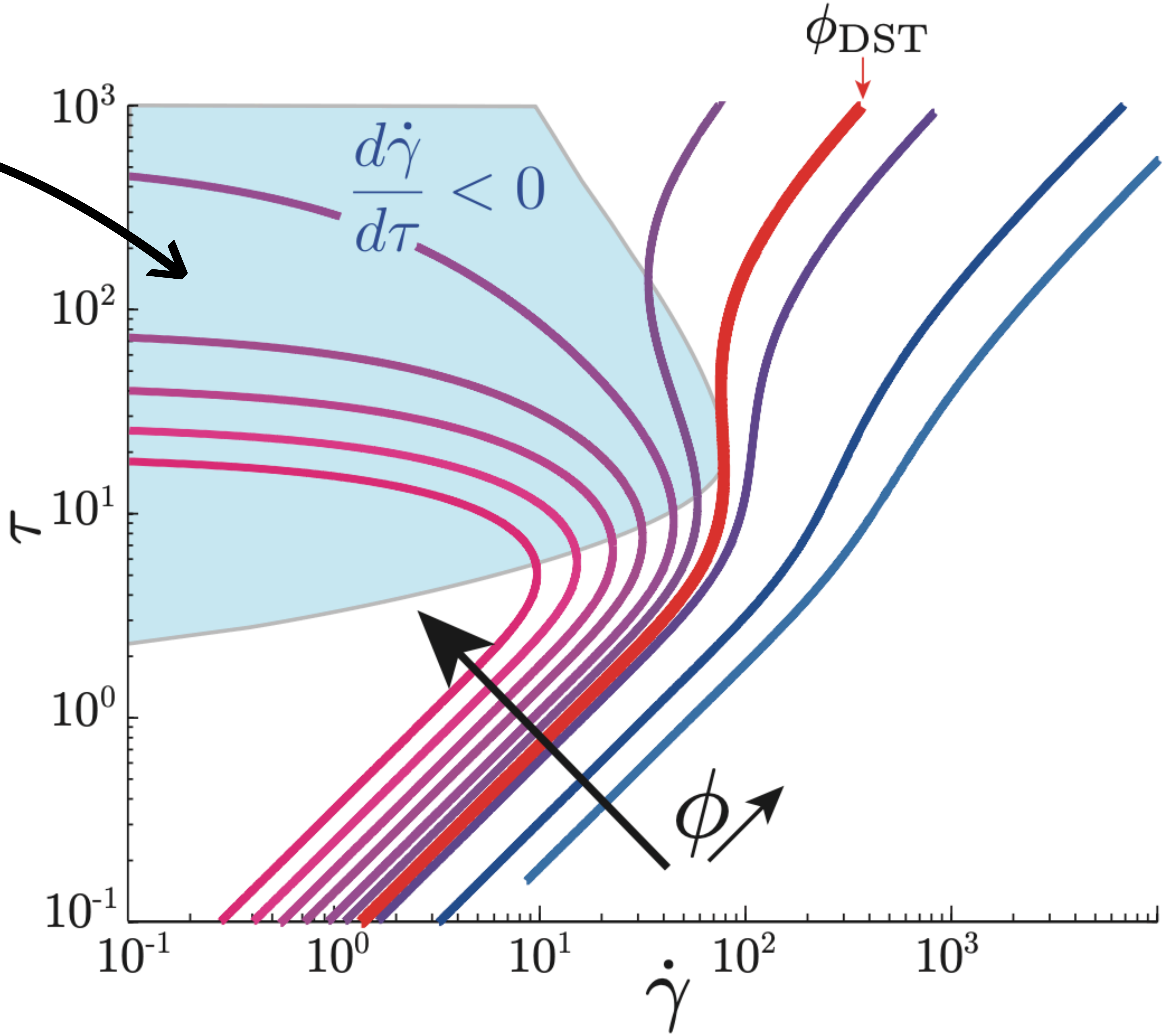


	$P_\mathrm{p}/P^\star$	Solvent	Frictional state
🔵	<<1	H_2O	Frictionless  P_p
🟧	>>1	H_2O	Frictional 
🔴	>>1	$H_2O+NaCl$	Frictional 
▽	>>1 to <<1	H_2O	Frictional to Frictionless  P_p

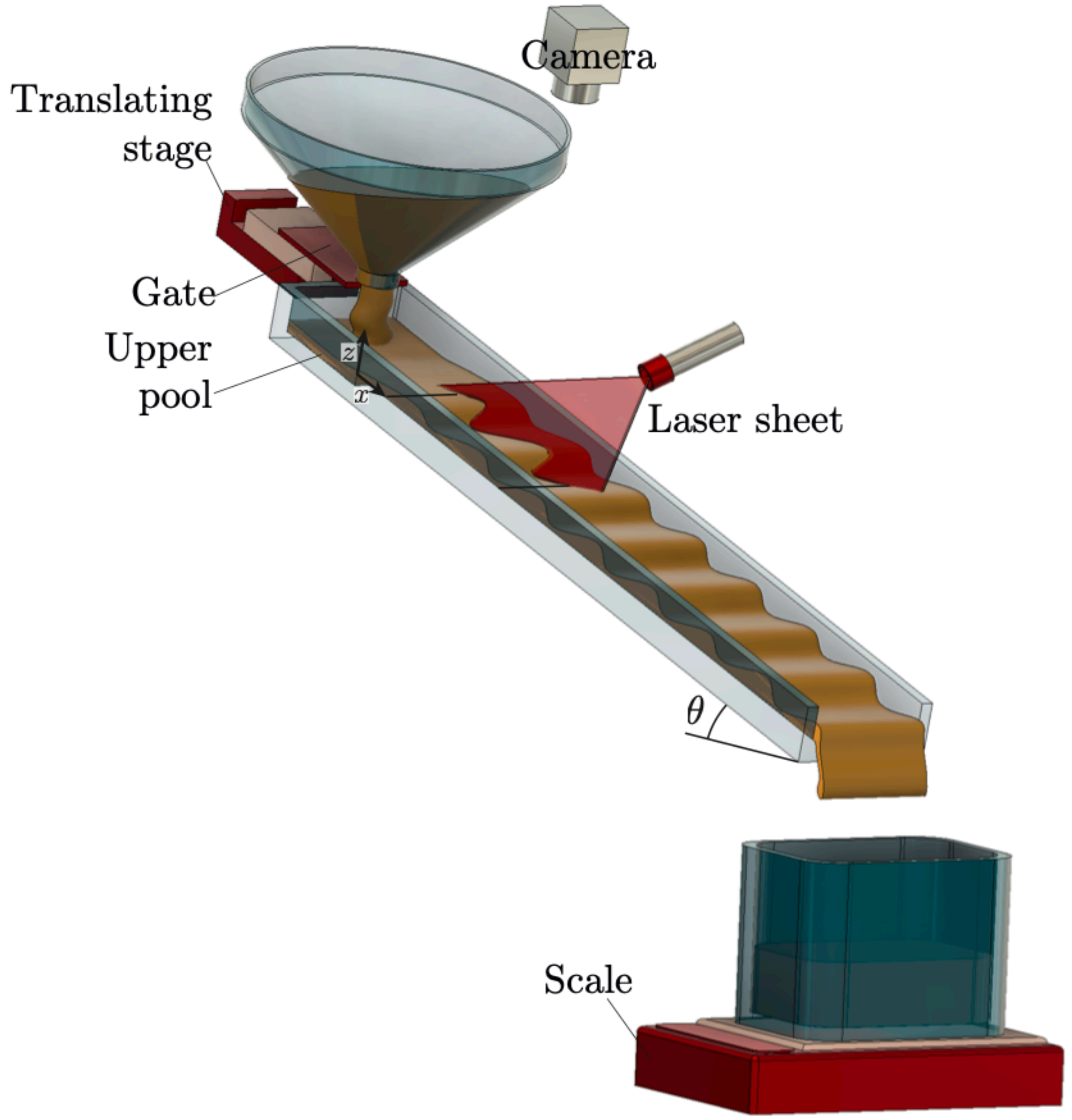
	Experiment	W&C Model
$\frac{P_p}{P^\star}$ <<1	$\mu = 0.13 + 1.7J^{0.3}$ → Frictionless particles!	 Frictionless
$\frac{P_p}{P^\star}$ >>1	$\mu = 0.3 + 3.2J^{0.5}$ → Frictional particles!	 Frictional
$\cancel{\frac{P_p}{P^\star}}$	Frictional transition	$\frac{P_p}{P^\star}$

Hydrodynamics of Shear—thickening Suspensions

Consequences of this velocity—weakening *[unstable]* branch to hydrodynamics?



Shear—thickening Suspensions down inclines: Oobleck Waves



Not understood, no flow rules !

(Balmforth Phys. Let. A 2005)

Darbois—Texier et al. Comm. Phys. (2020)

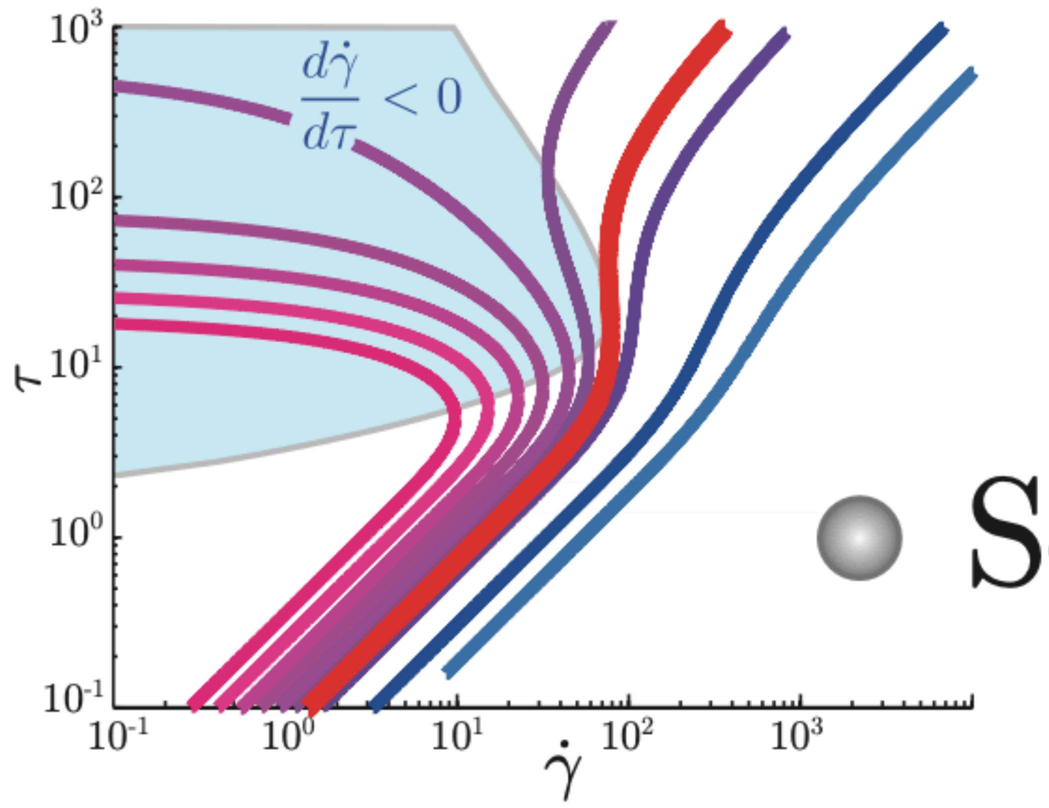
Darbois—Texier et al. JFM (2023)

Kapitza Waves in Water

Inertial Instability!

$$Re_c = Re_K$$

Instability Mechanism *without* Inertia



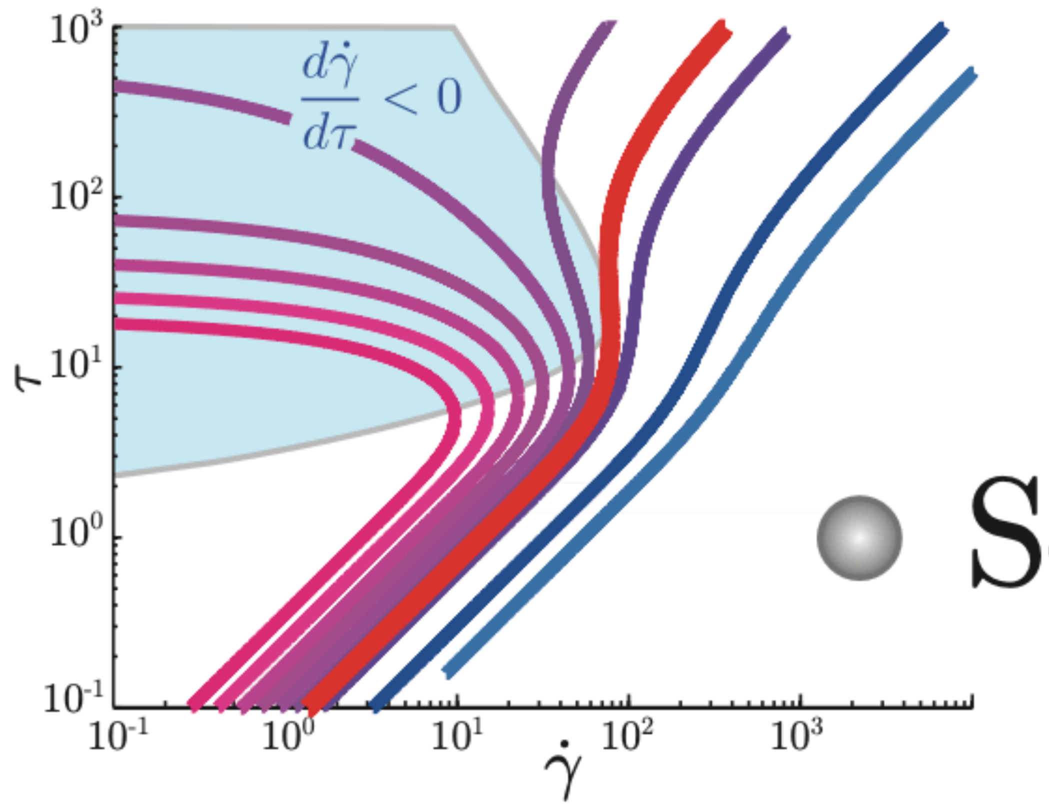
- S-shaped $A = \dfrac{d\dot{\gamma}}{d\tau} < 0$
- No inertia $Re \ll 1$

Force balance (depth averaged):

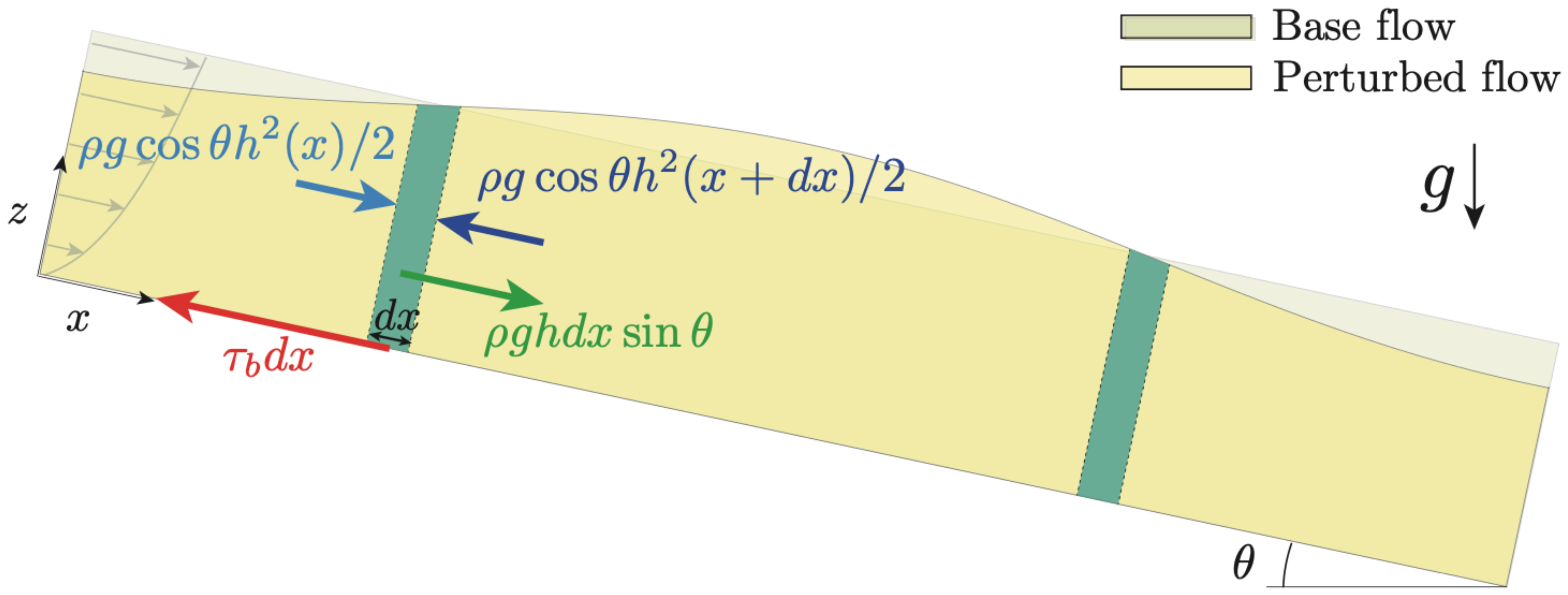
$$\tau_b = \rho g h \sin\theta$$

Basal stress Gravity

Instability Mechanism *without* Inertia



- S-shaped $A = \dfrac{d\dot{\gamma}}{d\tau} < 0$
- No inertia $Re \ll 1$



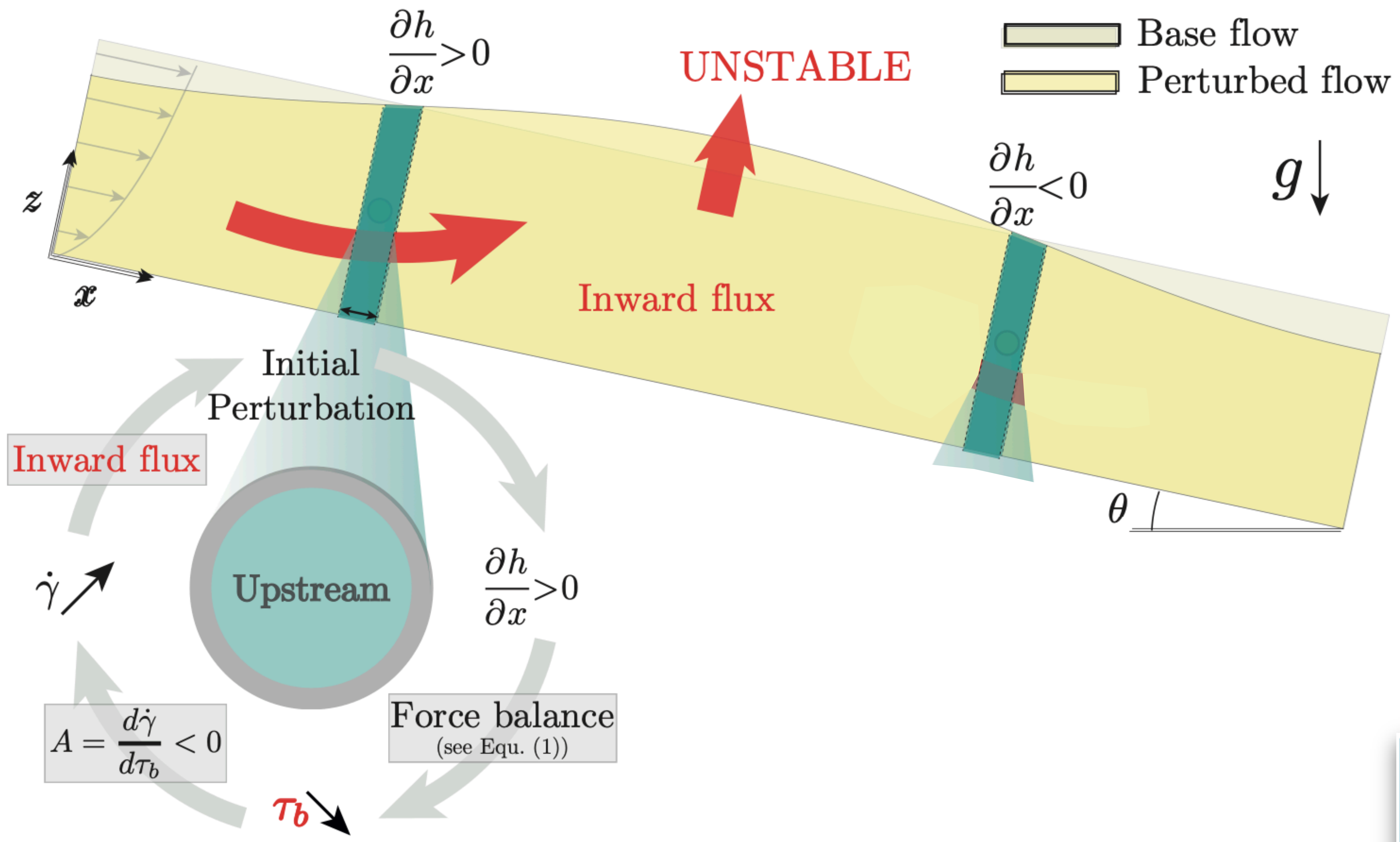
Force balance (depth averaged):

$$\tau_b = \rho g h \sin\theta - \rho g h \cos\theta \frac{\partial h}{\partial x}$$

Basal stress Gravity Hydrostatic pressure

Instability Mechanism *without* Inertia

- S-shaped $A = \frac{d\dot{\gamma}}{d\tau} < 0$
- No inertia $Re \ll 1$



Force balance (depth averaged):

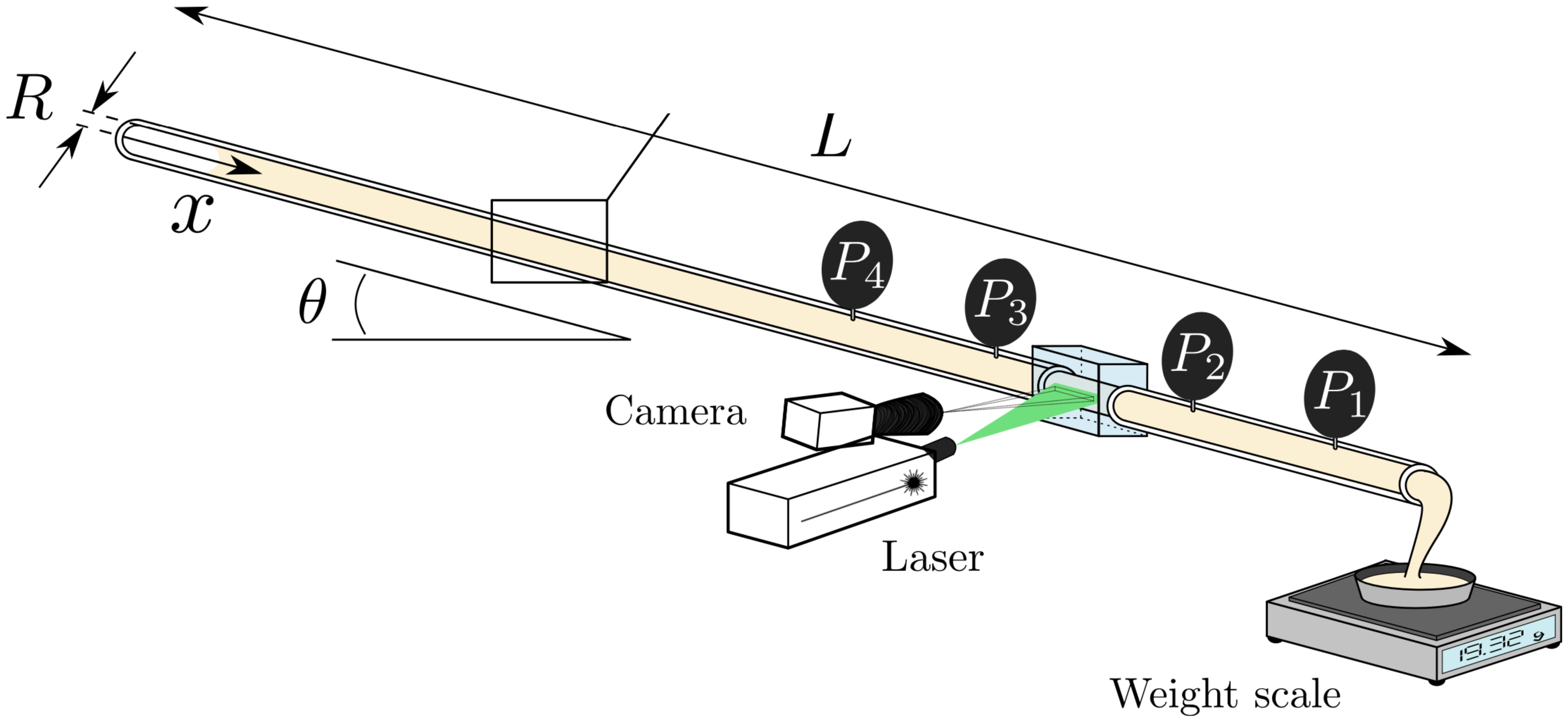
$$\tau_b = \rho g h \sin\theta - \rho g h \cos\theta \frac{\partial h}{\partial x}$$

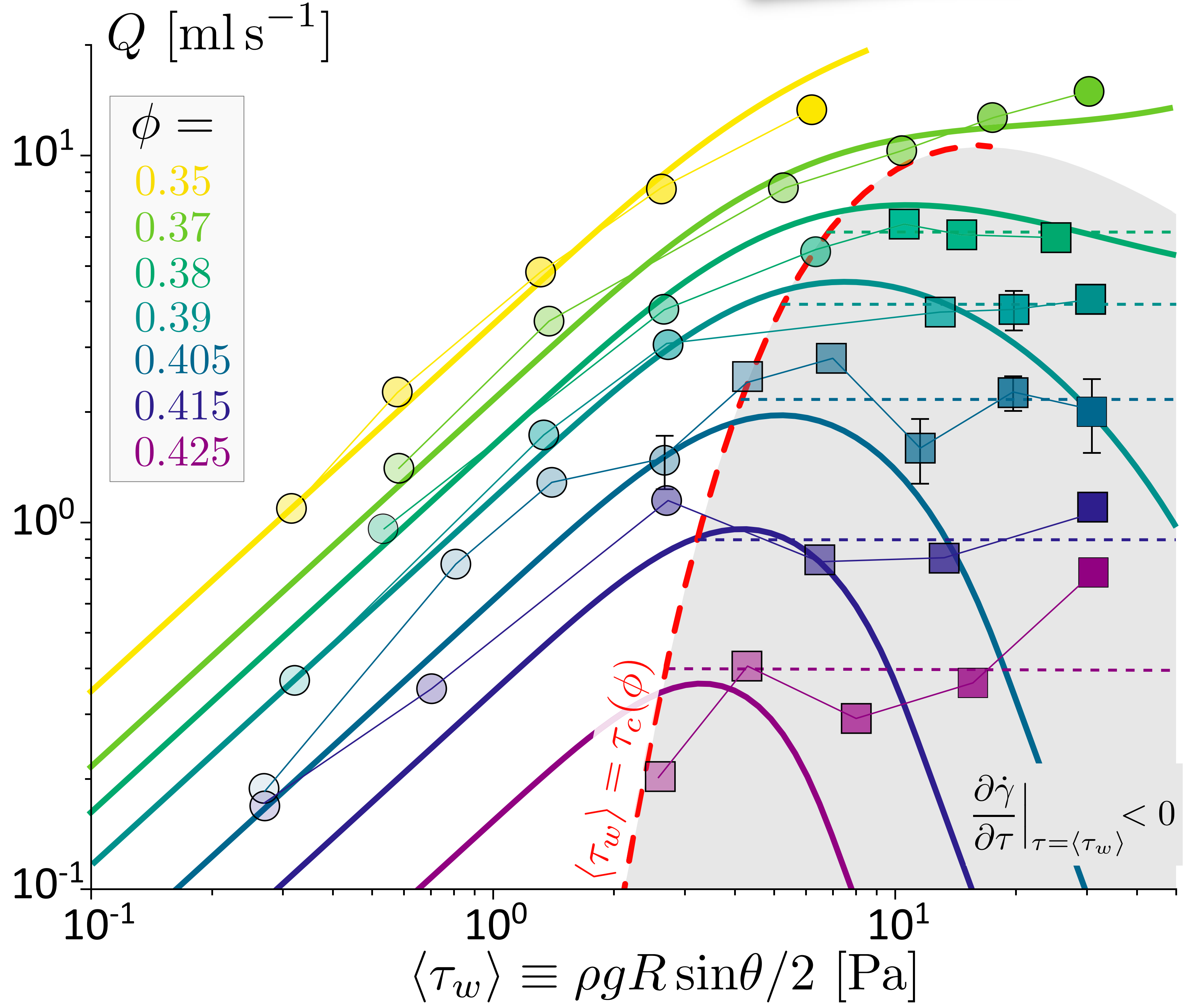
Basal stress Gravity Hydrostatic pressure

$\rightarrow$ Unstable if $A = \frac{d\dot{\gamma}}{d\tau} < 0$

Pipe flow of Shear—thickening Suspensions

Talk:
Alexis Bougouin
S2 on Friday

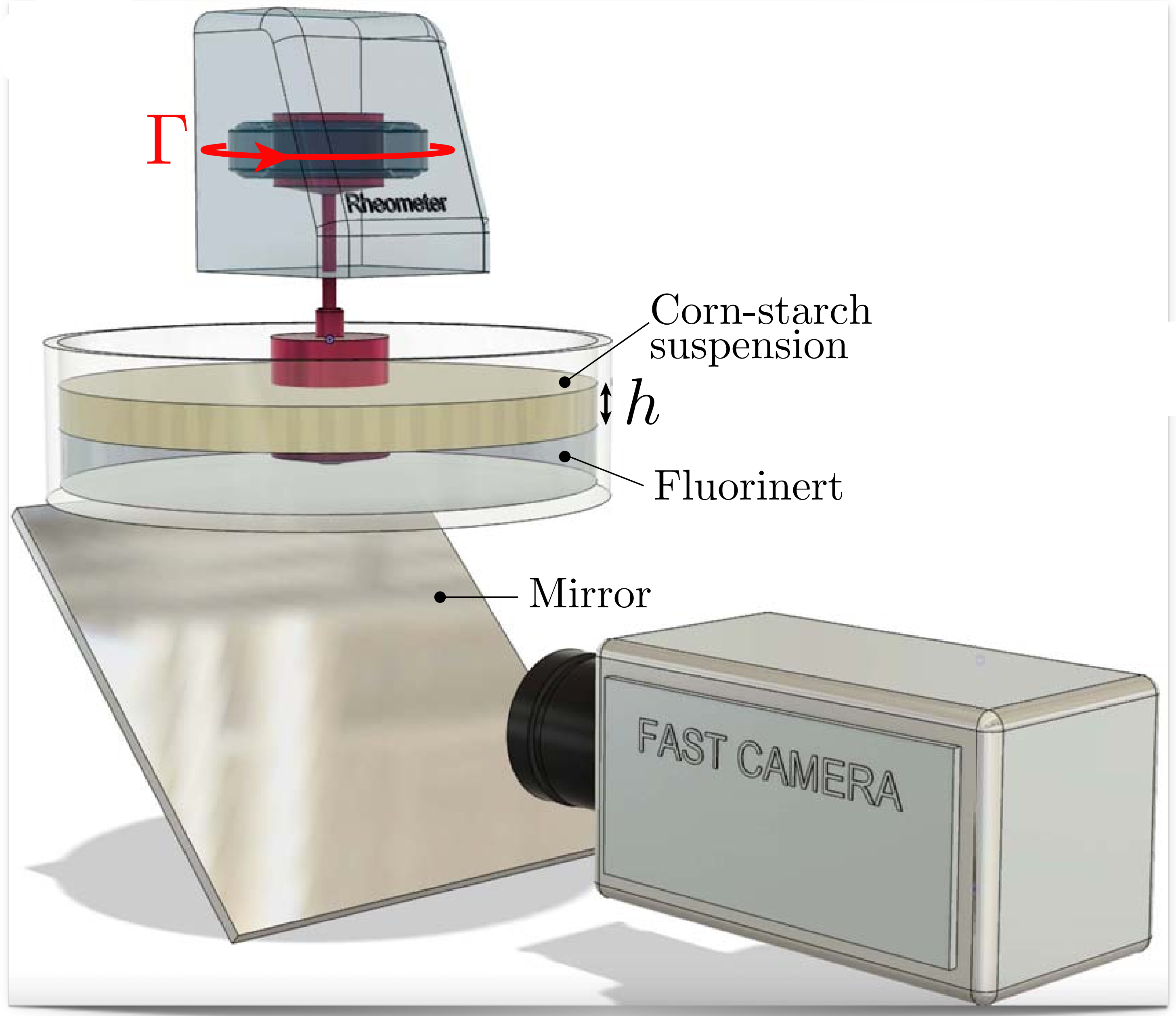


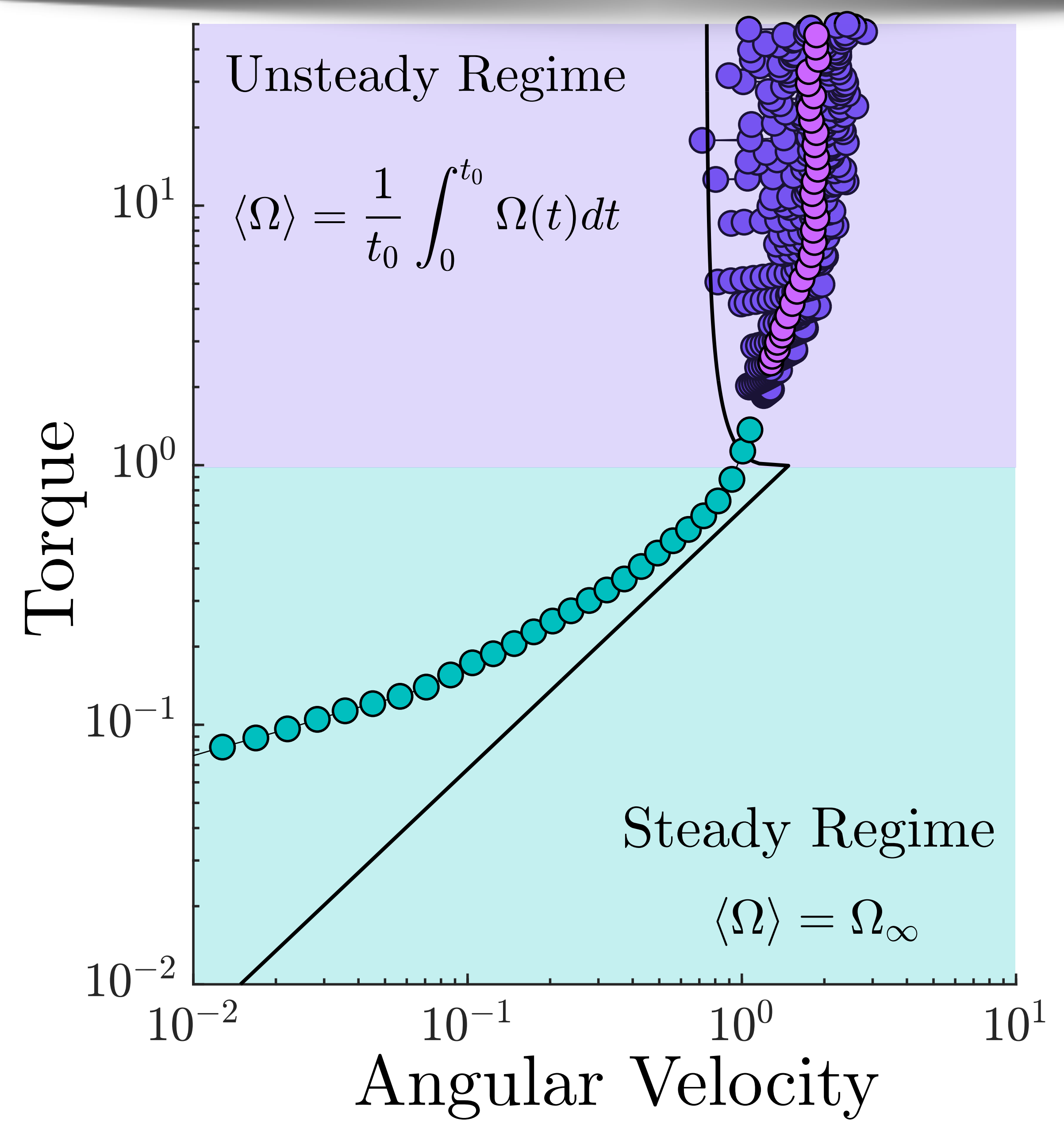


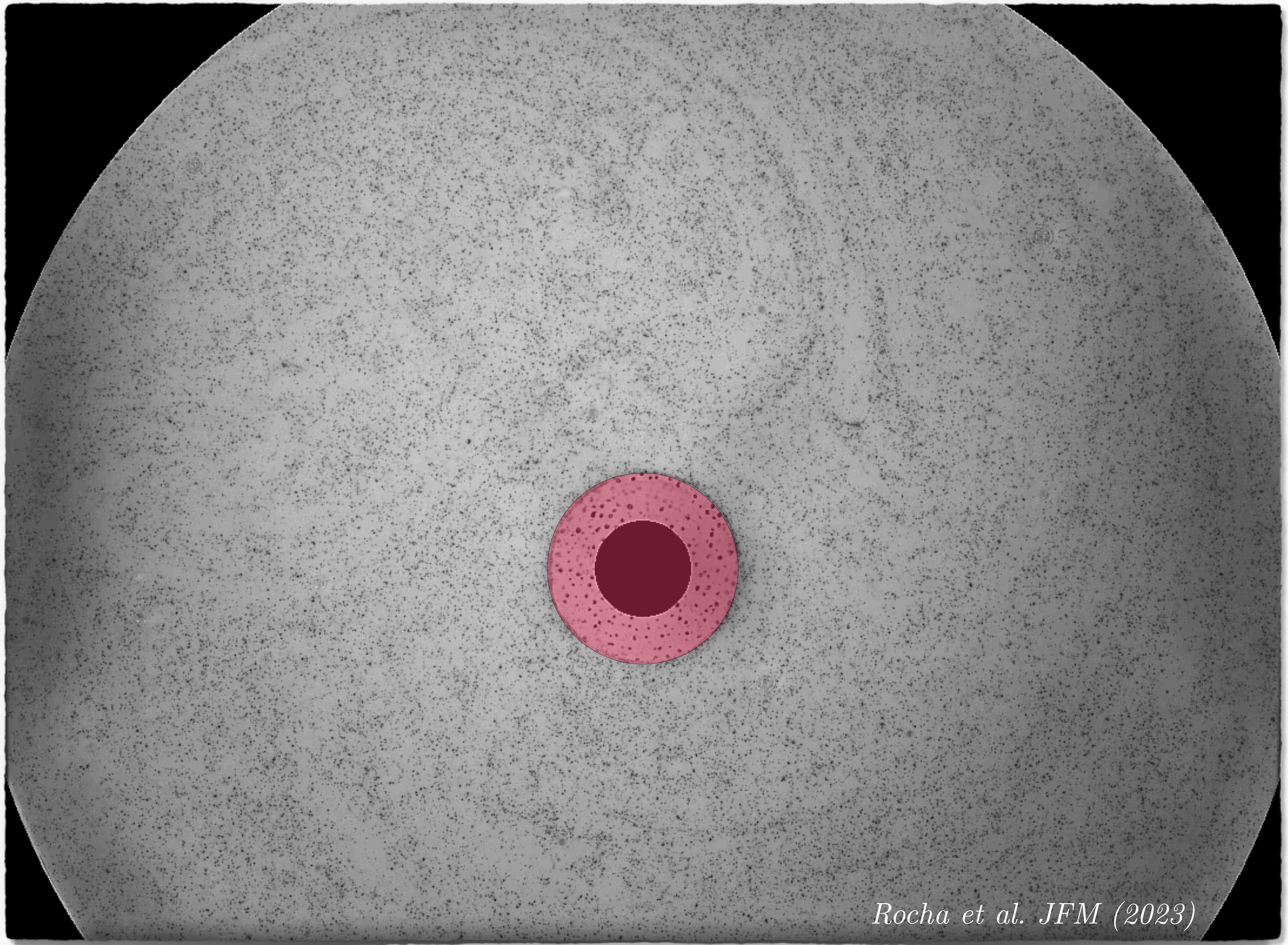
A retrogressive <u>FRICTIONAL SOLITARY WAVE</u> controls the flow rate!

Bougouin et al. PNAS (2024)

Drag Forces in Shear—thickening Suspensions

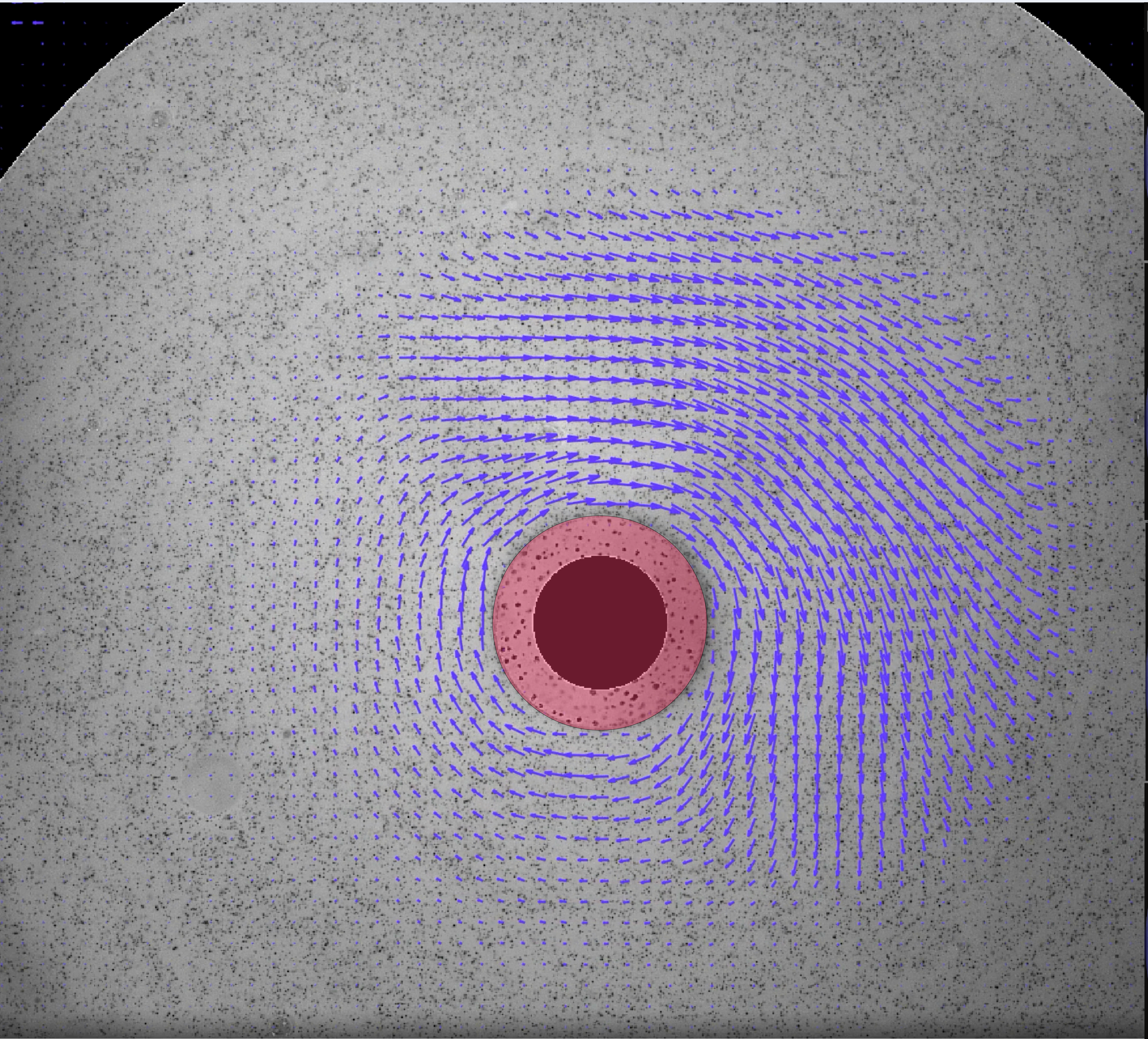


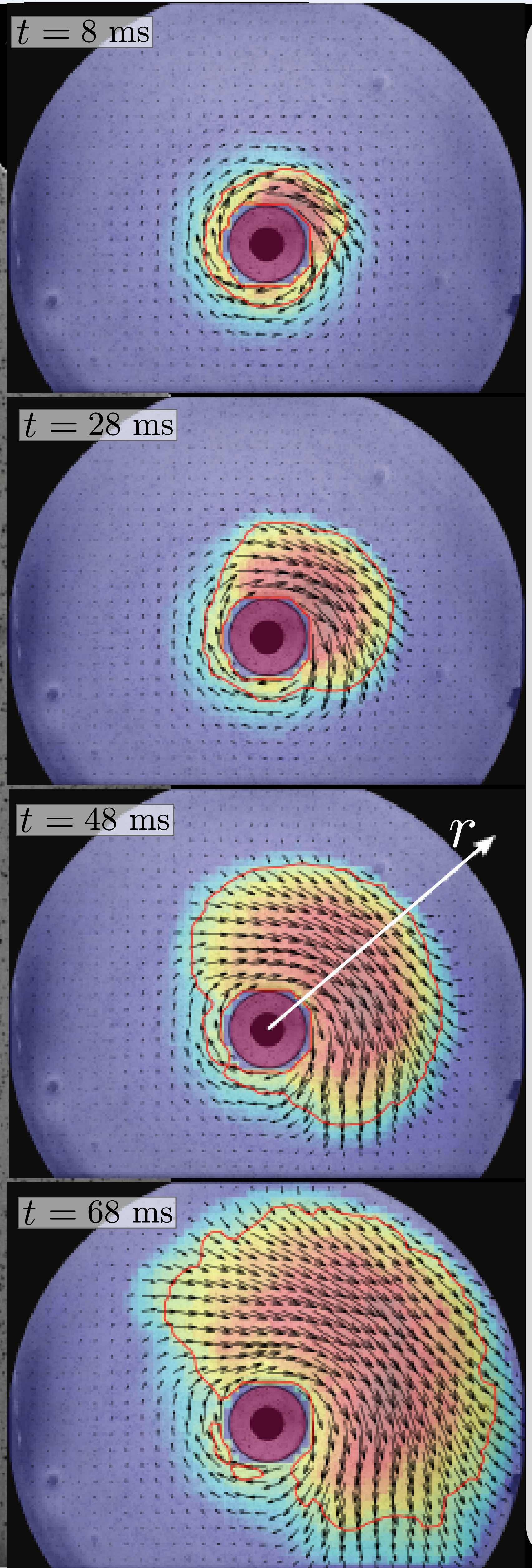


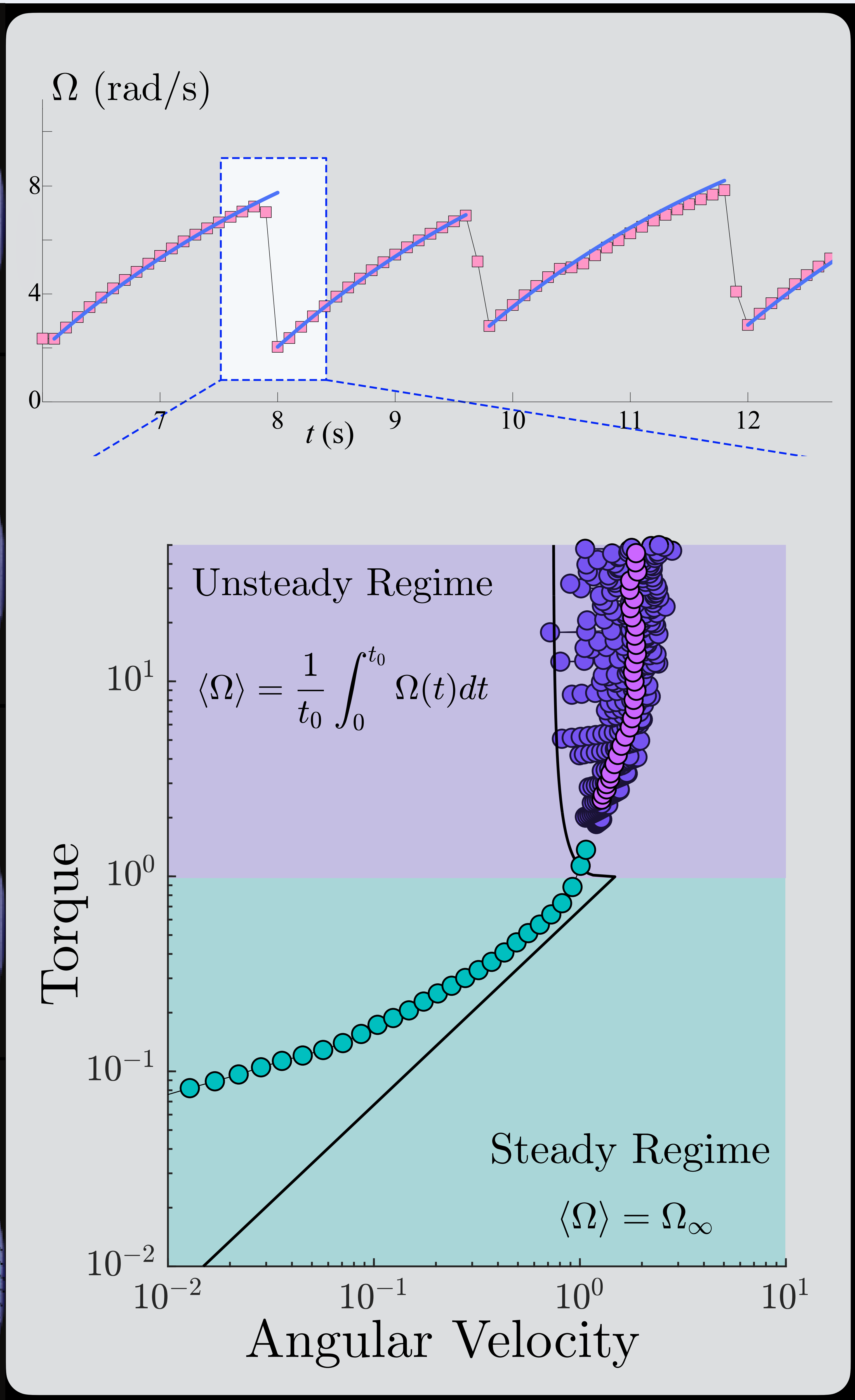


Rocha et al. JFM (2023)

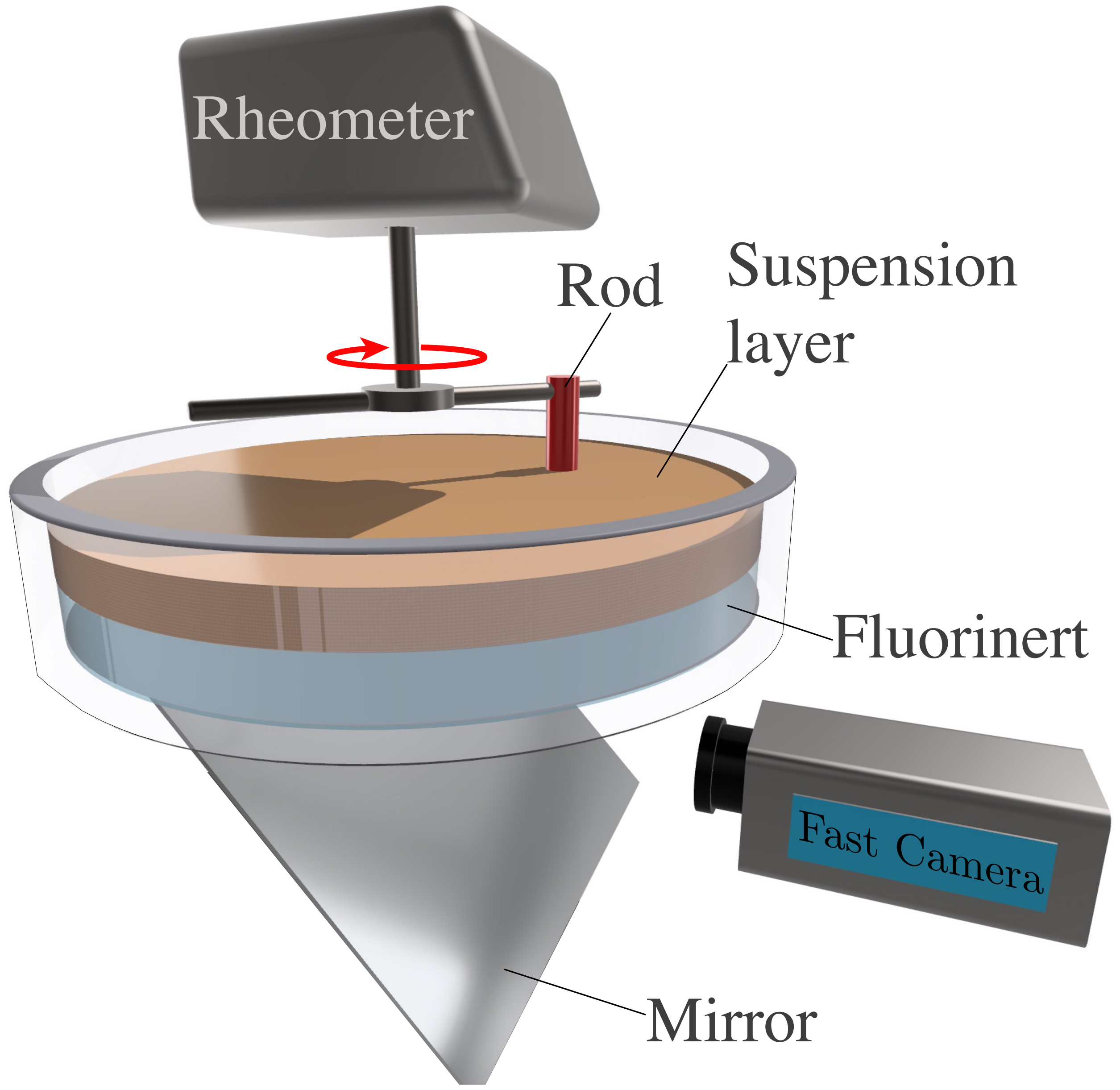
Drag Forces in Shear—thickening Suspensions

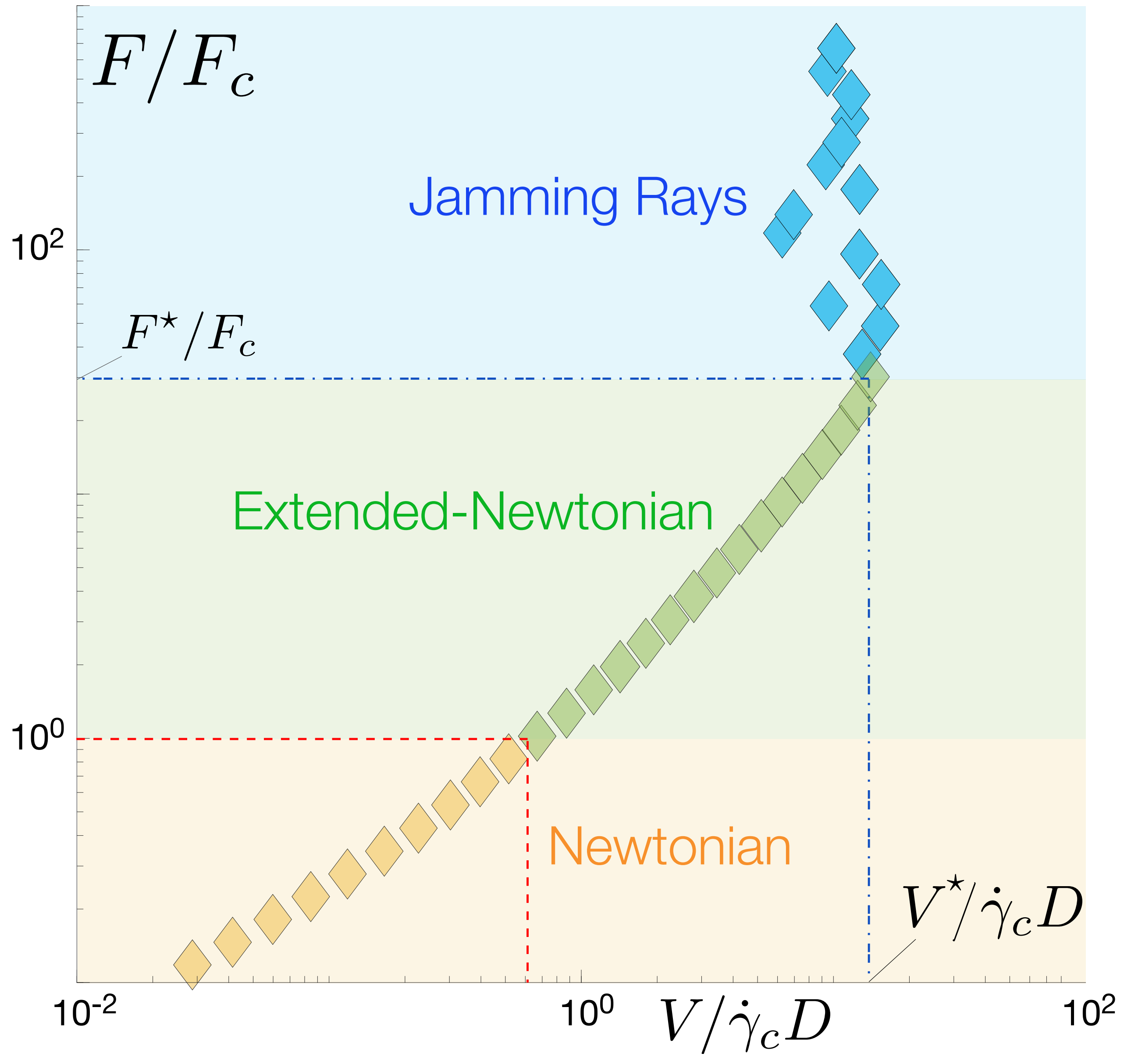






Flow of a Shear—thickening Suspensions around a Cylinder

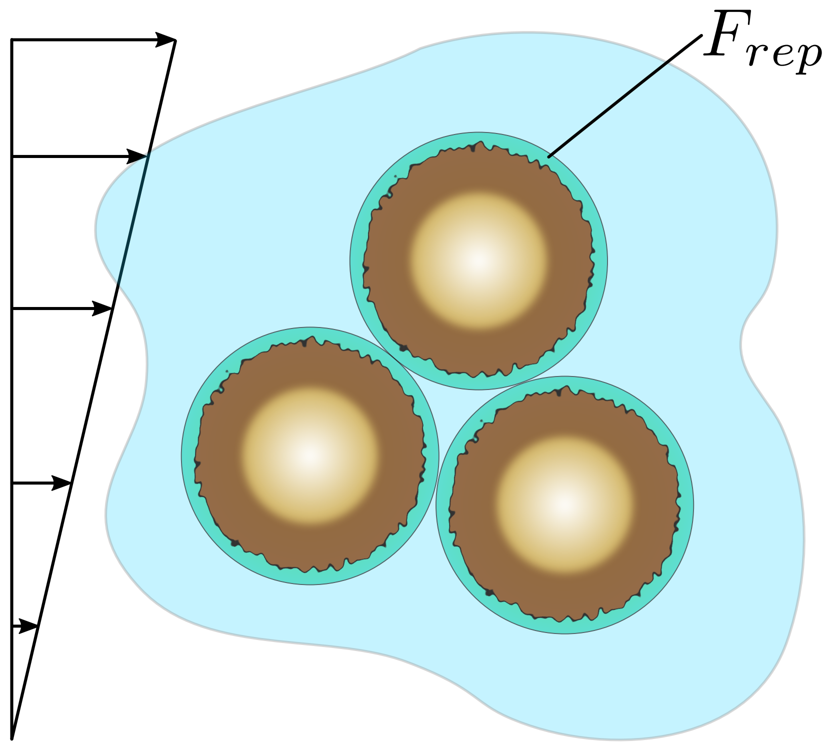




(Rocha et al. to be submitted)

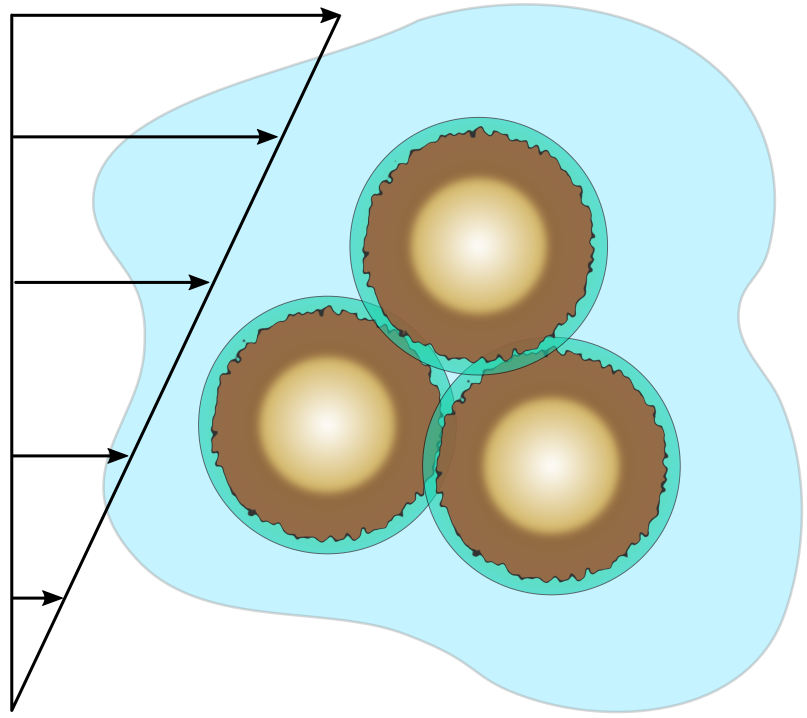
To Conclude...

Frictional Transition at the particle scale



Low Stress $P < P_{rep}$

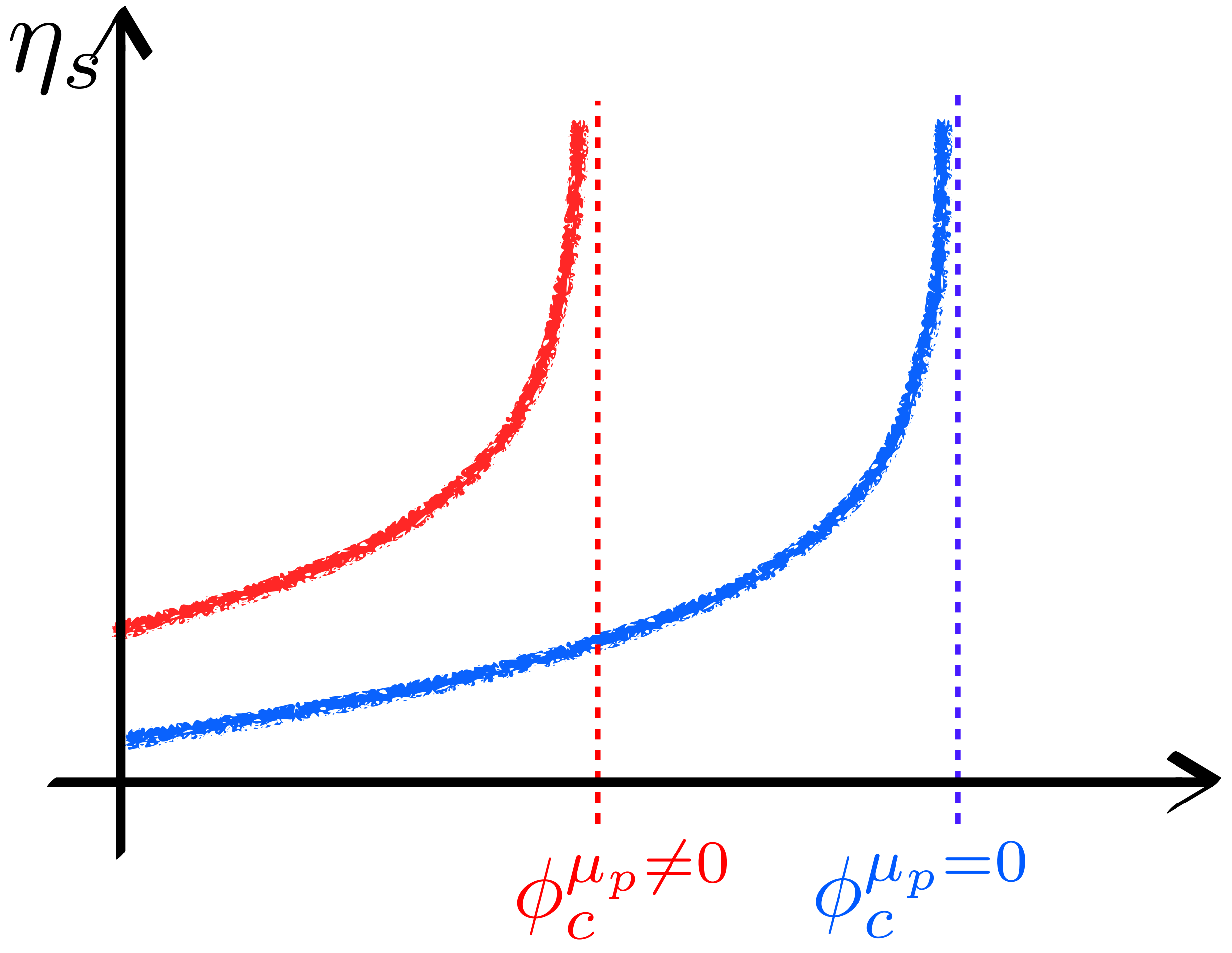
Frictionless

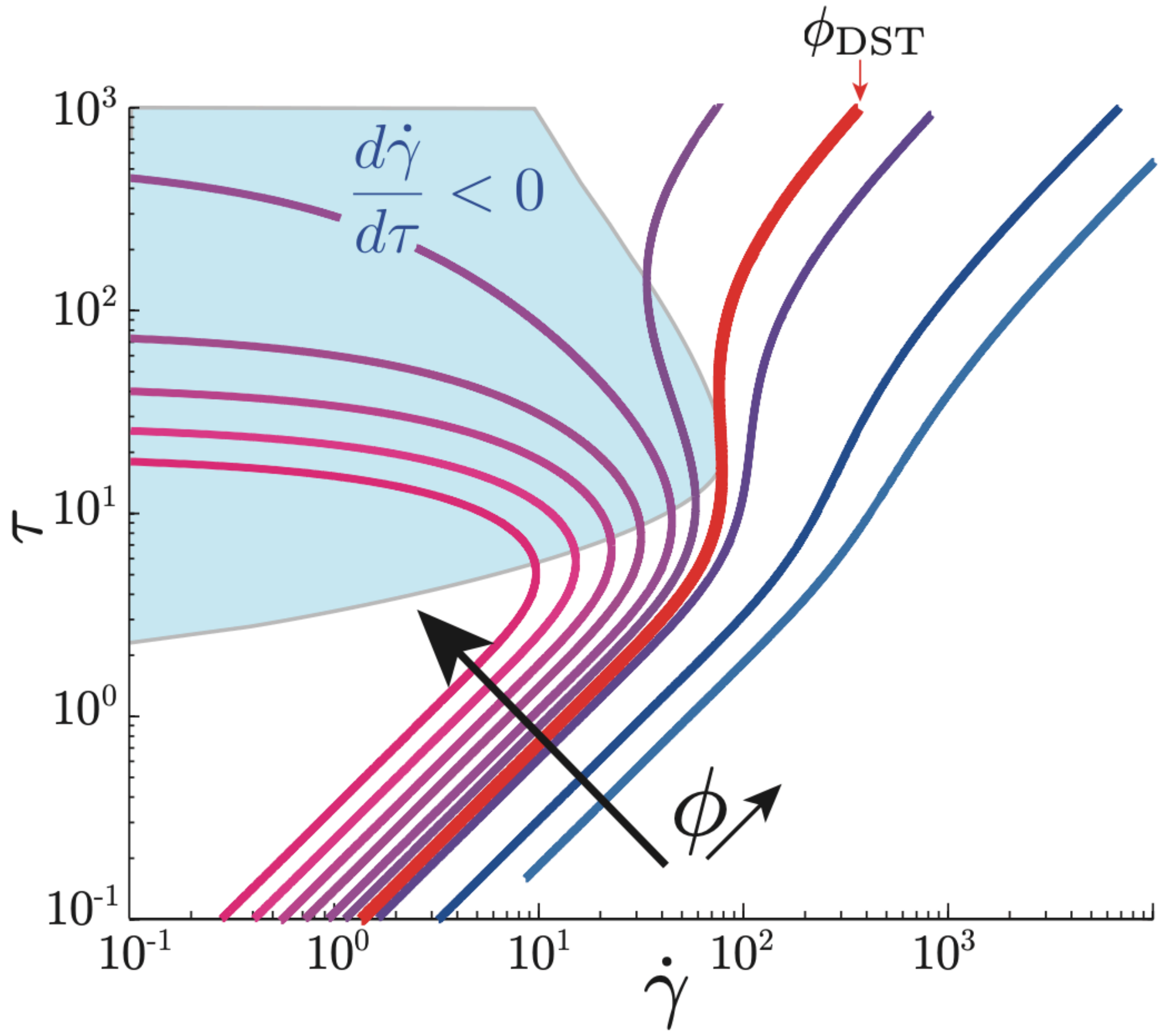


High Stress $P > P_{rep}$

Frictional

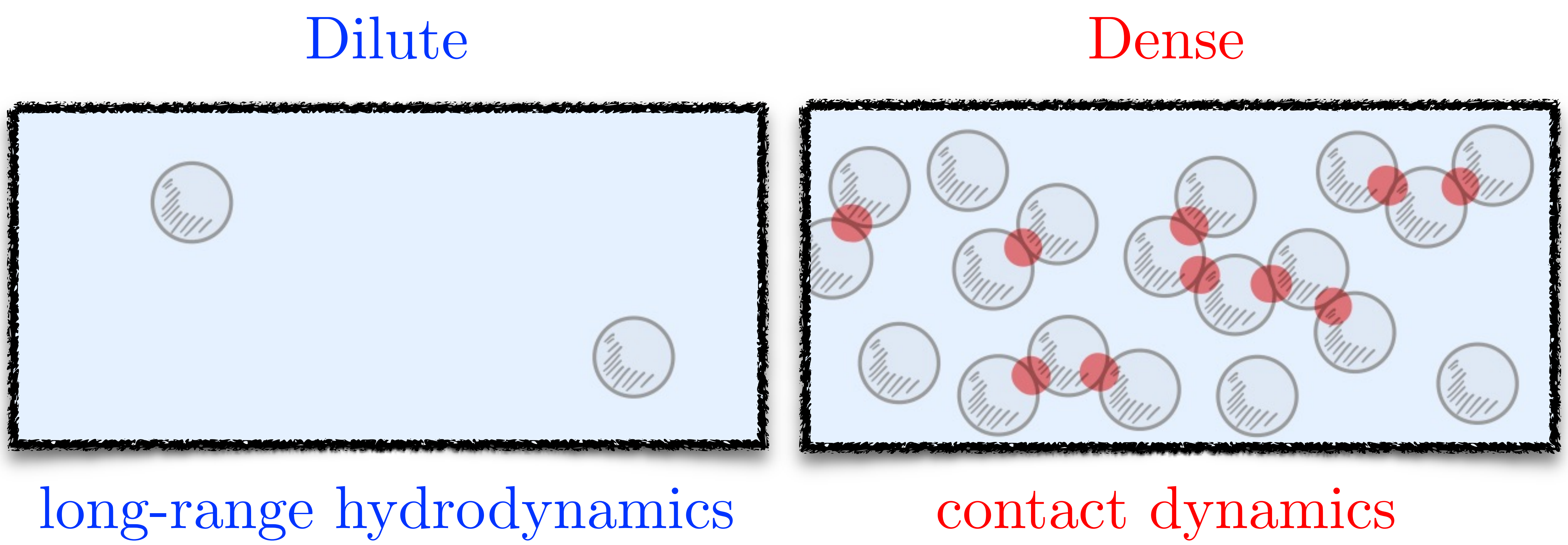
Peculiar Rheology [Flow Rules]

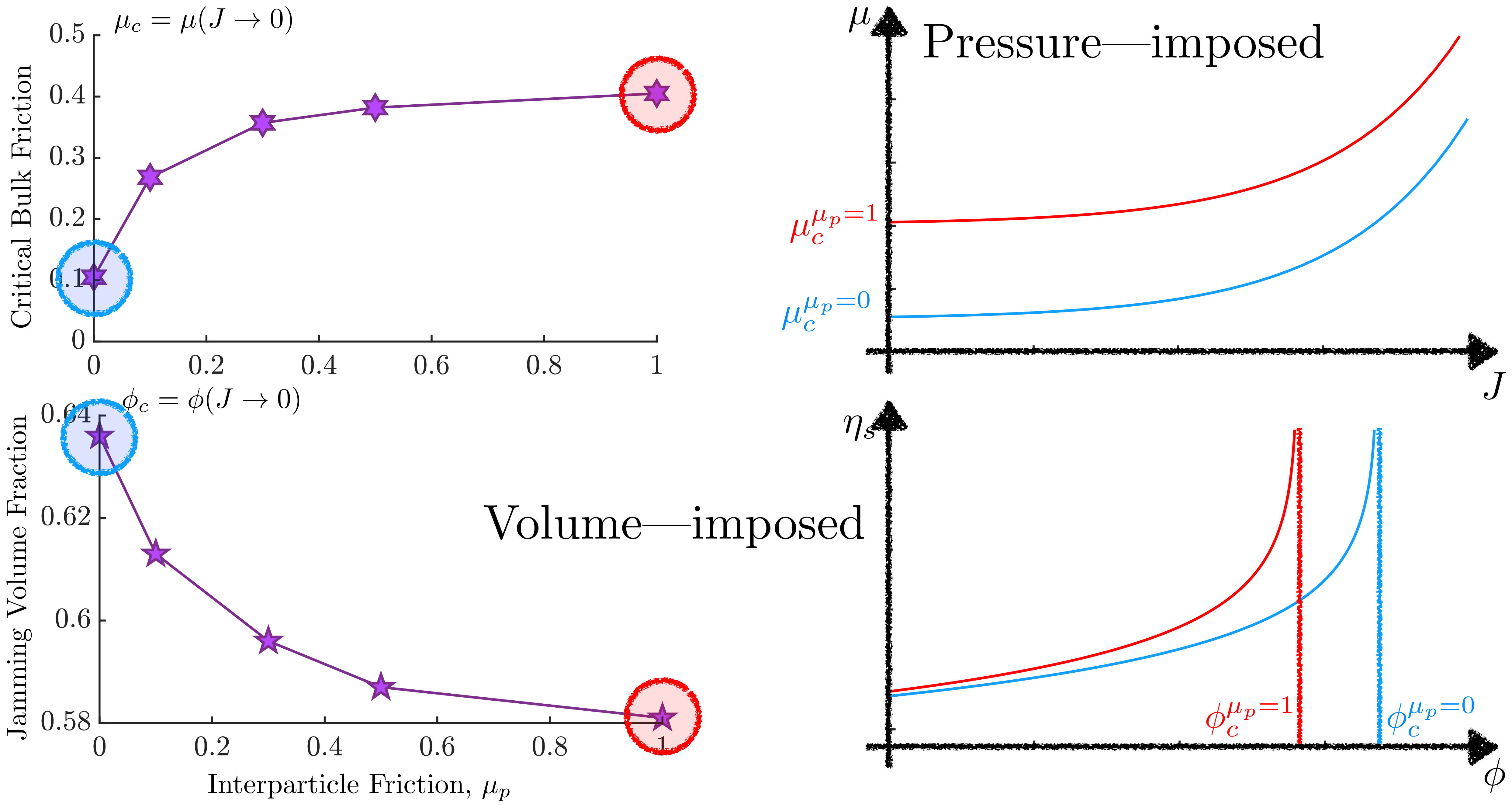




Flow Properties

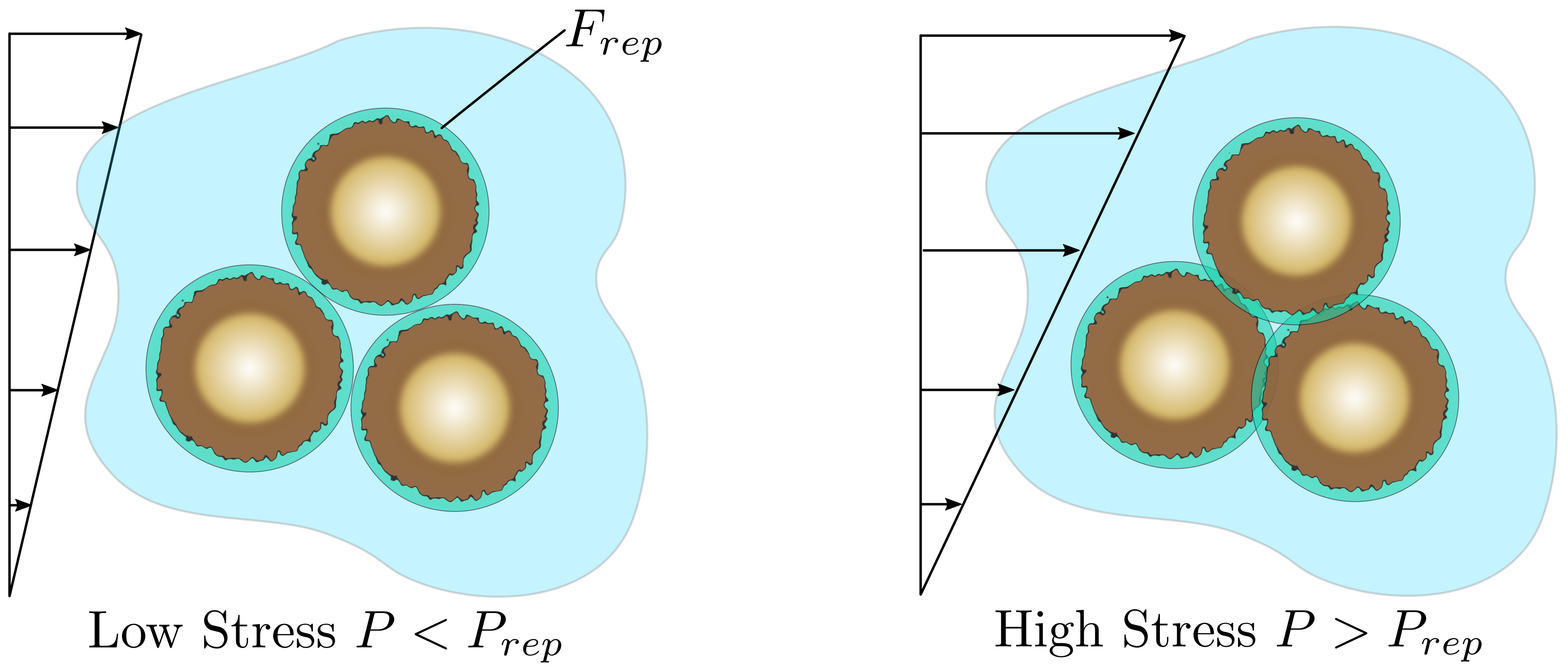
To Conclude...





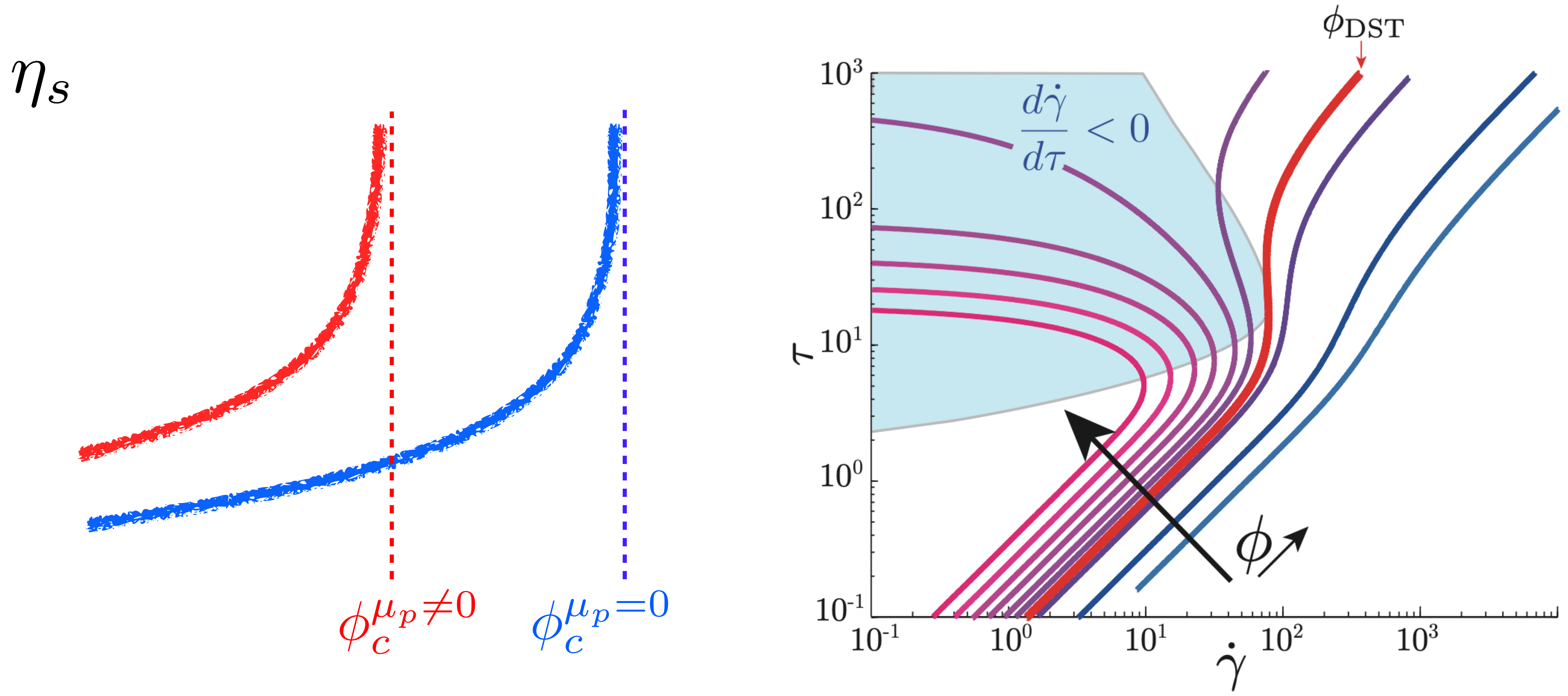
If φ_c changes dynamically → non-Newtonian Behaviours

e.g. Shear thickening



Frictionless

Frictional



** A couple of Reviews, Books & Nice Lectures **

1. E. Guazzelli & O. Pouliquen, JFM Perspectives (2018)
2. C. Ness, R. Seto & R. Mari, Annu. Rev. Condens. Matter Phys. (2022)
3. E. Guazzelli, Physical Review Fluids (2024)
4. J. Morris, Annu. Rev. Fluid Mech. (2020)
5. E. Guazzelli & J. Morris, Cambridge Press (2012)
6. B. Andreotti, Y. Forterre & O. Pouliquen, Cambridge Press (2004)
7. Elisabeth Lemaire — The Hitchhikers' of Rheology {Youtube}
8. Mike Cates — Trinity College Science Society {Youtube}
9. Wilson Poon — KITP Program: Physics of Dense Suspensions {KITP website}
10. Yoel Forterre — Département de Physique de l'ENS {Youtube}
11. Elisabeth Guazzelli — Australasian Fluid Mechanics Society {Youtube}

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